A problem from section

Kalina

February 5, 2015

Question 1 *Let A be a 4×3 non-zero matrix. And let $\vec{v}$ and $\vec{w}$ be vectors in $\mathbb{R}^3$ such that $A\vec{v} = A\vec{w}$. Does this imply that $\vec{v} = \vec{w}$?*

Answer: No. Here is an example of such a matrix with rank 2.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and take $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$.

Then $A\vec{v} = A\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.

Note however that this cannot be done if A has rank 3. Think why :)