PROBLEM SET 1
due February 14, 2017

1. Formulate and prove the Fundamental Theorem of Algebra in the tropical setting. Why is the tropical semiring algebraically closed.
2. Find all roots of the quintic $x^5 \oplus 1 \odot x^4 \oplus 3 \odot x^3 \oplus 6 \odot x^2 \oplus 10 \odot x \oplus 15$.
3. Show that the residue field $\Bbbk\{\{t\}\}$ is isomorphic to $\Bbbk$.
4. Let $K = \mathbb{C}\{\{t\}\}$ and consider the homogeneous polynomial

   $$f = (x_0 + t^{11}x_1 + t^{38}x_2)^{2000} + (x_0x_1 + t^{-9}x_0x_2 + t^{13}x_1x_2)^{1000}.$$

   Determine $trop(f)(\boldsymbol{w})$ for $\boldsymbol{w} = (3, 4, 5)$ and $\boldsymbol{w} = (30, 40, 50)$.
5. The quotient ring $K = \mathbb{Q}[s]/(3s^3 + s^2 + 36s + 162)$ is a field. Describe all valuations on this field that extend the 3-adic valuation on $\mathbb{Q}$.
6. (bonus) What is the maximal number of facets of any four-dimensional polytope with eight vertices?