PROBLEM SET 1
due January 24, 2018

Solve at least three problems:

1. Show that a vector space is a semi-simple as a module over the base field.
2. Let R be a finite dimensional F-algebra, where F is a field. Show that the following are equivalent:
   (a) R is a division ring (skewfield)
   (b) R is non-trivial with no zero divisors.
3. Show that the center of a simple ring is a field. What is the center of $M_n(D)$ for a division ring D?
4. Let
$$R = \left\{ \begin{bmatrix} b & 0 \\ a & c \end{bmatrix} : a, b \in \mathbb{R}, c \in \mathbb{Q} \right\}.$$
   Which is true for R: right/left Artinian, right/left Noetherian.
5. Let $R = M_n(D)$ for a division ring D. For any $1 \leq i \leq n$ define I_i to be those matrices, where every column other than the i^{th} is zero. Verify that these are minimal left ideals of R, isomorphic to each other when regarded as left R-modules.
6. Show that every simple left R-module is isomorphic to a factor module of ${}_R R$.
7. Let n be a positive integer, F a field of characteristic not dividing n and Z_n the cyclic group of order n. Describe the ideals of the group algebra FZ_n.
8. Determine the projective modules of $\mathbb{Z}/(6)$.
9. Show that a module M has a composition series if and only if it is both Artinian and Noetherian.