PROBLEM SET 3
due February 18, 2018

Solve at least three problems:

1. Prove that the character table of G is an invertible matrix.
2. Compute the character of the group $\mathbb{Z}_2 \times \mathbb{Z}_4$.
3. Show that the character table of the dihedral group D_4 of order 8 is the same as the character table of the group of quaternions Q_8.
4. Prove that if χ_U is a linear character of $\mathbb{C}G$ and χ_V is an irreducible character of $\mathbb{C}G$ the the product $\chi_U \chi_V$ is an irreducible character of $\mathbb{C}G$.
5. Show that the orthogonal central idempotents of $\mathbb{C}G$ can be computed using the formula
$$e_i = \frac{1}{|G|} \sum_{g \in G} \chi_i(1)\chi_i(g^{-1})g.$$
6. Let N be a normal subgroup of G, and let V be a $\mathbb{C}(G/N)$-module. Show that if ϕ is the character of V, then when V is viewed as a $\mathbb{C}G$-module the corresponding character is $\phi\pi$, where $\pi : G \to G/N$ is the natural projection.
7. Find the character table of S_4.(Hint: Use the previous exercise)