PROBLEM SET 4
due March 7, 2018

Solve at least three problems:

1. Find infinitely many distinct primary decompositions of $(x^2, xy) \subseteq \mathbb{R}[x, y]$.
2. Find the primary decomposition of $(x^2 - (y+1)^3, (y^2 - 1)^2) \subseteq \mathbb{Q}[x, y]$. (Warning: This requires patience and determination!)
3. Find the primary decomposition of $(x^2 - yz, x(z - 1)) \subseteq \mathbb{Q}[x, y, z]$.
4. Let I be an ideal of R and $x \in R$ such that both $I + (x)$ and $(I : x)$ are finitely generated. Show that I is also finitely generated.
5. Show that if the nilradical of R is trivial then the Jacobson radical of $R[x]$ is trivial. (Hint: find a condition for the invertibility of a polynomial)
6. Show that if the nilradical is a maximal ideal then (0) is a primary ideal.
7. Let I, J be two ideals of a Noetherian ring. Show that there exists an integer n such that $I \cap J^n \subseteq IJ$.