PROBLEM SET 2
due March 5, 2020

Solve at least three problems:

1. Show that if A, B are nilpotent ideals of R then so is their sum $A + B$.
2. Show that if R is a ring then $J(R)$ contains every nil left ideal.
3. Let R be a J-semisimple domain (that is $J(R) = 0$ and there are no zero divisors). Let $a \in Z(R) \setminus 0$. Conclude that the maximal left modules not containing a have a trivial intersection.
4. Assume $I_1, I_2, \ldots I_n$ are ideals of R such that R/I_i are all semisimple. Show that $R/\cap I_i$ is also semisimple. (Hint: One can assume that they are all maximal and distinct).
5. Give a counterexample to the above claim if there are infinitely many ideals.
6. Is it true that the minimal left ideals of $M_n(R)$ are of the form $M_n(I)$, where I is a minimal left ideal?
7. We know that R is a ring with exactly 7 non-trivial left ideals. Show that one of them is two-sided.
8. Are there non-trivial units of $\mathbb{Z}C_7$, where C_7 is the cyclic group of order 7? (The trivial units of a group ring $\mathbb{Z}G$ are of the form $\pm g$ with $g \in G$).