PROBLEM SET 1
due September 13, 2018

1. Compute the remainder on division of the polynomial $f = x^7y^2 + x^3y^2 - y + 1$ by the polynomials $g = xy^2 - x, h = x - y^3$. Use the graded lexicographical order.

2. Show that there is a unique monomial order on $\mathbb{C}[x]$.

3. Suppose that $I = \left\langle \boldsymbol{x}^{\alpha(1)}, \dots, \boldsymbol{x}^{\alpha(s)} \right\rangle \subseteq k[x_1, \dots, x_n]$ is a monomial ideal. Prove that a polynomial f is in I if and only if the remainder of f on division by $\{\boldsymbol{x}^{\alpha(1)}, \dots, \boldsymbol{x}^{\alpha(s)}\}$ is zero.

4. Let $I = \left\langle z - x^2, y - x^3 \right\rangle$ be an ideal of $\mathbb{C}[x, y, z]$. Use Buchberger's Criterion to check if $G = \{z - x^2, y - x^3\}$ is a Gröbner basis for I with respect to the lexicographic order.

5. Let $I \in k[x_1, \dots, x_n]$ be a principal ideal. Show that any finite subset of I containing a generator for I is a Gröbner basis for I.

6. Let $f \in k[x_1, \dots, x_n]$. If $f \notin \langle x_1, \dots, x_n \rangle$, then show that $\langle f, x_1, \dots, x_n \rangle = k[x_1, \dots, x_n]$.