

PROBLEM SET 3
due October 23, 2018

Notation and conventions:

We assume that varieties are irreducible.

1. Show that the quasi-projective variety $\mathbb{A}^2 \setminus \{0, 0\}$ is not isomorphic to an affine variety.
2. Let $H = V(F) \subset \mathbb{P}^n$ be a projective hypersurface defined by one homogeneous polynomial $F \in k[x_0, \dots, x_n]$ of degree d . Show that $\mathbb{P}^n \setminus H$ is isomorphic to an affine variety. (Hint: Understand first the case $d = 1$).
3. Prove that the image of the Veronese embedding is not contained in any hyperplane of \mathbb{P}^n . (Hint: Understand what are the defining equations for the image of the Veronese embedding).
4. Prove that every rational map $\phi : \mathbb{P}^1 \rightarrow \mathbb{P}^n$ is regular.