PROBLEM SET 4
due December 4, 2018

Notation and conventions:
We assume that varieties are irreducible.

1. Determine the singular points on the Steiner surface in $\mathbb{P}^3$ with equation
$$x_1^2x_2^2 + x_0^2x_2^2 + x_0^2x_1^2 - x_0x_1x_2x_3 = 0.$$
2. Show that the product of smooth quasi projective varieties is smooth. (Hint: Use the Segre embedding to reduce to the affine case).
3. Show that a finite map between affine varieties is surjective. (Hint: You probably will need to use Nakayama's Lemma at some point).
4. Find the normalization of the plane curve defined by $x^2 + x^3 = y^2$.
5. Let $S \subseteq \mathbb{P}^3$ be a smooth cubic surface and let L_1 ad L_2 be two disjoint lines on S. Given a point $P \in \mathbb{P}^3 \setminus (L_1 \cup L_2)$ check that there is a unique line in $\mathbb{P}^3$, passing through P and intersecting L_1 and L_2. Denote by P_1 and P_2 for the intersection points. Explain that $P \mapsto (P_1, P_2)$ defines a rational map $S \to L_1 \times L_2$. (One can also show that there is a rational map $L_1 \times L_2 \to S$ making S is birational to $\mathbb{P}^1 \times \mathbb{P}^1$).