Approximate reasoning in stochastic concurrency

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Outline of Talk

- A quick tour of Hybrid cc: a synchronous concurrent constraint language for simulation and modelling of hybrid systems.
- Probabilities as an abstraction mechanism.
- Approximate reasoning with probabilities

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Hybrid CC

- (With Bobrow, Carlsson, Gupta, Saraswat).
- A synchronous concurrent constraint language for simulation and modelling of hybrid systems.
- Concurrent Constraint programming is shared memory computing where the shared memory is a constraint store.
- Information accumulates monotonically in the shared constraint store.

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Hybrid CC: Continuous Constraint System

- Continuous constraint system = constraint system extended over (real) time.
- There is no implicit carryover of information between time instants.
- Point phases and interval phases.

$$\bullet(\longrightarrow)\bullet(\longrightarrow)\bullet(\longrightarrow)\bullet(\longrightarrow)\bullet$$

- Point phases: based on constraints from prior interval phase (written $^{-}(\cdot)$) and constraints at this time instant
- Interval phases: evolution by an “integration” with initial values set up by prior point phase.

Hybrid CC: Program Combinators

- Structure: Parallel composition, Hiding
- Instantaneous primitives: `tell` c, `if` c `then` P, `if` c `else` P
- Extension over time: `always` P

A one dimensional “billiards” table

$$\begin{aligned}\mathtt{main} :: \ & \mathtt{always}[min = -5, max = 5], \mathtt{dyn}_b(b1), \mathtt{dyn}_b(b2), \\ & \mathtt{2coll}, \mathtt{pocket}\end{aligned}$$

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$$\begin{aligned} \mathtt{main} :: \quad & \mathtt{always}[max = -5, min = 5], \mathtt{dyn}_b(b1), \mathtt{dyn}_b(b2),\\ & \mathtt{2coll}, \mathtt{pocket} \end{aligned}$$

$$\begin{aligned} \mathtt{dyn}_b(B) :: \quad & \mathtt{always}\; ball(B),\\ & \mathtt{do}\\ & \quad \mathtt{always}\; \frac{d(B{:}pos)}{dt} = (B : vel),\\ & \qquad\qquad\quad \mathtt{default}(\frac{d(B{:}vel)}{dt} = 0)\\ & \mathtt{watching}\; pocketed(B) \end{aligned}$$

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```
main ::       always[$min = -5, max = 5$], dyn$_b$($b1$), dyn$_b$($b2$),
              2coll, pocket

dyn$_b$($B$) :: always $ball(B)$,
              do
                 always $\frac{d(B:pos)}{dt} = (B : vel)$
                        default($\frac{d(B:vel)}{dt} = 0$)
              watching $pocketed(B)$

pocket ::     $(\forall\ ball(B))$
                 always
                    if $^{-}(B : pos) \notin [min, max]$
                    then pocketed($B$)
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```
main ::        always$[min = -5, max = 5]$, $\mathtt{dyn}_b(b1)$, $\mathtt{dyn}_b(b2)$,
               2coll, pocket

$\mathtt{dyn}_b(B)$ ::  always $ball(B)$,
               do
                  always $\frac{d(B:pos)}{dt} = (B : vel)$
                         default$(\frac{d(B:vel)}{dt} = 0)$
               watching $pocketed(B)$

pocket ::      $(\forall\, ball(B))$
                  always
                     if $^-(B : pos) \notin [min, max]$
                     then pocketed$(B)$

2coll ::       $(\forall ball(B1), ball(B2))$
               always
                  if $^-(B1 : pos) = {}^-(B2 : pos)$
                  then $B1 : vel =^- (B2 : vel), B2 : vel =^- (B1 : vel)$
```

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Some experiments with Hybrid cc

- VRCC: A virtual reality modelling language: In Constraints project at INRIA (Codognet).
- Electromechanical systems: in model based computing groups at Xerox (Carlsson, Gupta, Saraswat, Struss) and NASA (Gupta, Sweet). Probabilities of failure incorporated into models (Williams, Seung Chung, Gupta, IJCAI 2001) to reason about “likely” causes.
- Biological systems: in ModBio project at LORIA (Bockmayr, Courtois, ICSB 2001). Probability arises in the modeling of kinetics in biochemical reactions or as stochastic noise used as an abstraction to specify incomplete knowledge.

Rest of Talk: Reasoning with Probabilities

- Exact reasoning: eg. Weak bisimulation.
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  - Instability of “exact” equivalences.
  - Probability numbers are to be viewed as coming with some error estimate: reasoning principles based on the exact value of numbers are arguably of dubious value.
- A metric corresponding to weak bisimulation.
- Joint work with Desharnais, Gupta and Panangaden.

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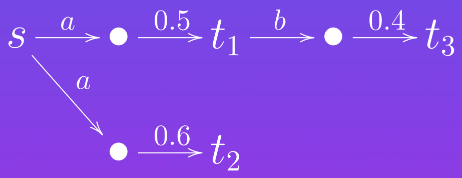
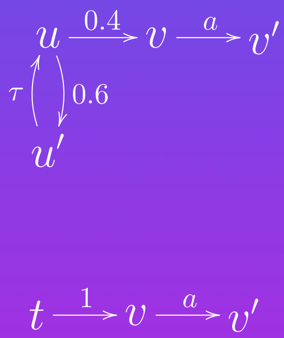
Labelled Concurrent Markov Chains

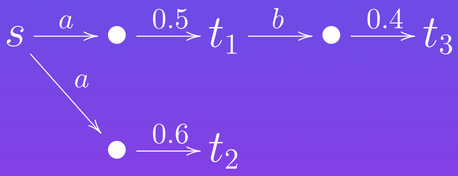
- Bipartite Graph: Probabilistic states and nondeterministic states
- Labelled Nondeterministic transitions. Probabilistic transitions are treated as having label τ.





Schedulers/Policies

- Complete resolution of nondeterminism to yield a labelled Markov chain



- For above, we get as possible schedules/computations, all prefixes of:

$$s \xrightarrow{a} \bullet \xrightarrow{0.5} t_1 \xrightarrow{b} \bullet \xrightarrow{0.4} t_3 \qquad\qquad s \xrightarrow{a} \bullet \xrightarrow{0.6} t_2$$

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Weighted combinations of Schedulers/Policies

$$t \xrightarrow{1} v \xrightarrow{a} v'$$

$$\begin{tikzcd}
u \arrow[r, "0.4"] \arrow[d, bend left, "0.6"] & v \arrow[r, "a"] & v' \\
u' \arrow[u, bend left, "\tau"] & &
\end{tikzcd}$$

$$(0.6)[t \overset{\tau}{\Longrightarrow} t] + (0.4)[t \overset{\tau}{\Longrightarrow} v]$$

$$\begin{tikzcd}
u \arrow[r, "0.4"] \arrow[d, bend left, "0.6"] & v \\
u' &
\end{tikzcd}$$

$$(0.36)[t \overset{\tau}{\Longrightarrow} t] + (0.64)[t \overset{\tau}{\Longrightarrow} v]$$

$$\begin{tikzcd}
u \arrow[r, "0.4"] \arrow[d, bend left, "0.6"] & v \\
u' \arrow[d, bend left, "\tau"] & \\
u \arrow[r, "0.4"] \arrow[d, bend left, "0.6"] & v \\
u' &
\end{tikzcd}$$

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Weak Bisimulation

- Let R be an equivalence relation. $s\ F(R)\ t$ if:

$$(\forall s \overset{a}{\Longrightarrow} P)\ (\exists t \overset{a}{\Longrightarrow} Q)\ \ P =_R Q$$

$$(\forall t \overset{a}{\Longrightarrow} Q)\ (\exists s \overset{a}{\Longrightarrow} P)\ \ P =_R Q$$

  where $P =_R Q$ if

$$(\forall R - \text{closed } E)\ P(E) = Q(E)$$

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A (pseudo)metric based approximate viewpoint

- Aim: Identify a metric on processes as a maximum fixed point.
- $\mathcal{M}$: 1-bounded pseudometrics on states with ordering

$$m_1 \preceq m_2 \text{ if } (\forall s,t)\ [m_1(s,t) \geq m_2(s,t)]$$

- $(\mathcal{M}, \preceq)$ is a complete lattice.

$$\begin{aligned}
\bot(s,t) &= \begin{cases} 0 \text{ if } s = t \\ 1 \text{ otherwise} \end{cases} \\
\top(s,t) &= 0, (\forall s,t) \\
(\sqcap\{m_i\}(s,t) &= \sup_i m_i(s,t)
\end{aligned}$$

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A (wannabe) maximum fixed point definition

- Let $m \in \mathcal{M}$. $F(m)(s,t) < \epsilon$ if:

$$(\forall s \overset{a}{\Longrightarrow} P)\ (\exists t \overset{a}{\Longrightarrow} Q)\ \ m(P,Q) < \epsilon$$

$$(\forall t \overset{a}{\Longrightarrow} Q)\ (\exists s \overset{a}{\Longrightarrow} P)\ \ m(P,Q) < \epsilon$$

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Hutchinson metric on probability measures

- $$d(\mu,\nu) = \sup_f \left| \int f\,d\mu - \int f\,d\nu \right|, f \text{ 1-Lipschitz}$$
- $m \in \mathcal{M}$. Let P, Q be probability distributions. Then:

$$m(P,Q) = \max \sum_i (P(s_i) - Q(s_i))a_i$$

subject to:

$$\begin{array}{l} \forall i. 0 \leq a_i \leq 1 \\ \forall i,j.\ a_i - a_j \leq m(s_i, s_j). \end{array}$$

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A functional on pseudo metrics

- Let $m \in \mathcal{M}$. $F(m)(s,t) < \epsilon$ if:

$$(\forall s \overset{a}{\Longrightarrow} P)\;(\exists t \overset{a}{\Longrightarrow} Q)\;\; m(P,Q) < \epsilon$$

$$(\forall t \overset{a}{\Longrightarrow} Q)\;(\exists s \overset{a}{\Longrightarrow} P)\;\; m(P,Q) < \epsilon$$

- The required metric on processes is the maximum fixed point of F: ie. the distance numbers are the least possible.

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Approximate reasoning is not harder!

- Bisimulation = metric distance 0.
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	Bisimulation	Metric
Logic	pCTL*	Real-valued modal logic
Compositionality	Congruence except +	Non-expansive except +
Proofs	Coinduction	Coinduction

$$\text{Non-Expansivity: } \frac{m(s,t) < \epsilon}{m(C[s], C[t]) < \epsilon}$$

- Duality theory of LP[1] plays a crucial role.

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[1] Explored in this context by van Breugel and Worrell.

Calculating quantitative observables

- Expectation of uniformly continuous functions is a continuous function of the metric.
- An example from information theory: channel capacity is a continuous function of the metric.

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Towards infinities

- Continuous state spaces: Tractable since probability bounds of 1 makes this case easier than full LP.
- Continuous time: preliminary results are encouraging.
- (Admittedly informal!) recipe to turn exact reasoning into approximate reasoning.

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