Recent Advances in Pure and Applied Stochastics

Titles and Abstracts

1. Fabrice Baudoin, University of Connecticut (fabrice.baudoin@uconn.edu)

   Title: Gundy-Varopoulos martingale transforms and their projection operators

   Abstract: We prove the L^p boundedness of generalized first order Riesz transforms obtained as conditional expectations of martingale transforms à la Gundy-Varopoulos for quite general diffusions on manifolds and vector bundles. Several specific examples and applications will be presented. This is joint work with Li Chen (University of Connecticut) and Rodrigo Banuelos (Purdue University).

2. Nawaf Bou-Rabee, Rudgers University (nawaf.bourabee@rutgers.edu)

   Title: Coupling for Hamiltonian Monte Carlo

   Abstract: We present a new coupling approach to study the convergence of the Hamiltonian Monte Carlo (HMC) method. Specifically, we prove that the transition step of HMC is contractive w.r.t. a carefully designed Kantorovich (L^1 Wasserstein) distance. The lower bound for the contraction rate is explicit. Global convexity of the potential is not required, and thus multimodal target distributions are included. Explicit quantitative bounds for the number of steps required to approximate the stationary distribution up to a given error ϵ are a direct consequence of contractivity. These bounds show that HMC can overcome diffusive behavior if the duration of the Hamiltonian dynamics is adjusted appropriately. This talk is based on joint work with Andreas Eberle and Raphael Zimmer.

3. Juraj Foldes, University of Virginia (foldes@virginia.edu)

   Title: TBA

   Abstract: TBA

4. David Herzog, Iowa State (dherzog@iastate.edu)

   Title: Ergodicity and Lyapunov functions for Langevin dynamics with singular potentials

   Abstract: We discuss Langevin dynamics of N particles on $\mathbb{R}^d$ interacting through a singular repulsive potential, e.g. the well-known Lennard-Jones type, and show that the system converges to the unique invariant Gibbs measure exponentially fast in a weighted total variation distance. The proof of the result turns on an explicit construction of a Lyapunov function. In contrast to previous results for such systems, our result implies geometric convergence to equilibrium starting from an essentially optimal family of initial distributions.

5. Gautam Iver, Carnegie Mellon University (gautam@math.cmu.edu)

   Title: Dissipation Enhancement by Mixing.

   Abstract: : We quantitatively study the interaction between diffusion and mixing in both the continuous, and discrete time setting. In discrete time, we consider a mixing dynamical system interposed with diffusion. In continuous time, we consider the advection diffusion equation where the flow of the advecting vector field is assumed to be sufficiently mixing. We explicitly estimate the dissipation time based on the mixing rate. Moreover, in the discrete time setting, we show that the L^2 energy decays double exponentially in time, and this double exponential rate is achieved for by a large class of toral automorphisms.

6. Sean Lawley, University of Utah (lawley@math.utah.edu)

   Title: Diffusion in a switching or diffusing environment

   Abstract: A number of diverse biological systems involve diffusion in a randomly switching or diffusing environment. For example, such processes arise in chemoreception, neural communication, and insect respiration. Mathematically, these systems can be modeled by PDEs with boundary conditions described by either jump processes or SDEs. In this talk, we will demonstrate that these systems (a) arise naturally in applications and (b) are mathematically rich. In particular, we will highlight the delicate and counterintuitive behavior that these systems can exhibit.

7. Vincent Martinez, Hunter College (vrmartinez@hunter.cuny.edu)

   Title: Asymptotic coupling and uniqueness of invariant measures for the damped, stochastically-driven Korteweg de Vries equation

   Abstract: In their 1967 seminal paper, Foias and Prodi captured precisely a notion of finitely many degrees of freedom in the context of the two-dimensional (2D) incompressible Navier-Stokes equations (NSE). In particular, they proved that if a sufficiently large spectral projection of the difference of two solutions converge to 0 asymptotically in time, then the corresponding complementary projection of their difference must also converge to 0 in the infinite-time limit. In other words, the high modes are “eventually enslaved“ by the low modes. One could thus define the number of degrees of freedom of the flow to be the smallest number of modes needed to guarantee this convergence for a given flow, insofar as it is represented by a solution to the NSE. This property has since led to several developments in the understanding and application of the long-time behavior of solutions to the NSE, for instance, in the context of turbulence, data assimilation, and the existence of determining forms. In this talk, we will discuss this asymptotic enslavement property as it regards the issue of uniqueness of invariant measures for stochastically forced equations, particularly the damped, stochastic KdV equation.

8. Jonathan Mattingly, Duke Unversity (jonm@math.duke.edu)

   Title: TBA

   Abstract: TBA

9. Hung Nguyen, Iowa State (dhung@iastate.edu)

   Title: The generalized Langevin equation with power-law memory

   Abstract: The generalized Langevin equation (GLE) is commonly used to describe the velocity of microparticles in viscoelastic fluids. Describing the long-term behavior of diffusive GLEs in non-linear potentials is a long-standing open problem. We will look at recent advances in establishing existence and uniqueness of a stationary distribution for an infinite-dimensional Markov representation of the GLE. If time permits, we will also discuss asymptotic behaviors of the GLE in different limits, namely, the small-mass limit and the white noise limit.

10. Georg Menz, University of California, Los Angeles (gmenz@math.ucla.edu)

    Title: Ergodicity of the infinite swapping algorithm at low temperature.

    Abstract: About a joint work with Andr Schlichting, Wenpin Tang, and Tianqi Wu. Sampling Gibbs measures at low temperature is a very important task but computationally very challenging. Numeric evidence suggest that the infinite-swapping algorithm (isa) is a promising method. The isa can be seen as an improvement of replica methods which are very popular. We rigorously analyze the ergodic properties of the isa in the low temperature regime and show that the effective energy barrier can be reduced drastically using the isa compared to the classical over-damped Langevin dynamics. As a consequence we obtain that sampling and simulated annealing is improved by an exponential factor.

11. Geordie Richards, Utah State (geordie.richards@usu.edu)

    Title: On Ergodicity and Stochastic Boussinesq Equations

    Abstract: We will review some recent results on applications of ergodic theory in the analysis of stochastic Boussinesq equations, which are model equations for Rayleigh-Bénard convection perturbed by an additive noise in temperature. First we will discuss applications of towards singular parameter limits, such as a proof that the Nusselt number, a measure of heat transfer, converges to a unique value in the infinite Prandtl number limit. Then we will discuss a computational approach to hydrodynamic stability, which also invokes ergodicity, when the stochastic forcing in temperature is restricted to the boundary of the domain. This talk is based on joint works with Juraj Foldes, Nathan Glatt-Holtz, and Jared Whitehead.

12. Samuel Punshon-Smith, Brown University (punshs@brown.edu)

    Title: Lagrangian chaos and scalar mixing in stochastic fluid mechanics

    Abstract: Lagrangian chaos refers to the chaotic behavior of Lagrangian trajectories in a fluid. This chaotic behavior often characterized by a positive Lyapunov exponent, namely the property that initially close trajectories will separate at an exponential rate after long time. In this talk we will consider a variety of stochastically forced fluid models, including the 2 dimensional Navier-Stokes equations, and show that under certain non-degeneracy conditions on the noise, the stochastic flow possesses a positive Lyapunov exponent. The proof crucially uses the theory of random dynamical systems as well as tools from Malliavin calculus and control theory to satisfy a certain non-degeneracy criterion originally due to Furstenberg. We will explore several important consequences of a positive exponent for passive scalars advected by the fluid. For instance, as a corollary it is straight forward to rigorously prove Yaglom's law, an analogue of Kolmogorov's famous 4/5 law for passive scalar turbulence. More importantly, using ideas from a paper by Baxendale and Stroock, one can show almost sure exponential decay in time of passive scalars in any negative Sobolev norm (exponential mixing) by using the positive exponent to show geometric ergodicity of the two point Lagrangian motion. Curiously, the proof of this currently uses both the strong Feller property and a Lasota-Yorke type gradient bound (or asymptotic strong Feller bound) for a certain degenerate Markov semi-group associated to the dynamics.

13. David Shirokoff, New Jersey Institute of Technology (david.g.shirokoff@njit.edu)

    Title: Convex relaxations for variational problems arising from models of self-assembly

    Abstract: We examine the problem of minimizing a class of nonlocal, nonconvex variational problems that arise from modeling a large number of pairwise interacting particles in the presence of thermal noise (i.e. molecular dynamics). Although finding and verifying local minima to these functionals is relatively straightforward, computing and verifying global minima is much more difficult. Global minima (ground states) are important as they characterize the structure of matter in models of self-assembly. We discuss how minimizing the functionals can be viewed as testing whether an associated bilinear form is co-positive. We then develop sufficient conditions for global optimality (which in some cases are provably sharp) obtained through a convex relaxation related to the cone of co-positive functionals. The advantage of the convex relaxation is that it results in a conic variational problem that is "computationally tractable", and may be solved using modern numerical techniques. The solutions provide fundamental information on the shapes of self-assembled materials in the corresponding models.

14. Gideon Simpson, Drexel University (grs53@drexel.edu)

    Title: Sampling on Rough Energy Landscapes

    Abstract: I will highlight the sampling challenges posed by rough energy landscapes, showing how a certain type of roughness impacts standard methods like MALA and RWM. Results and asymptotics explaining this performance will be presented and illustrated with numerical experiments. Methods for overcoming these obstacles will be discussed. This is joint work with Petr Plehac (University of Delaware).

15. Molei Tao, Georgia Tech (mtao@gatech.edu)

Title: A sampling challenge posed by a class of learning and inference tasks

Abstract: Additive gradients that sum a large amount of terms need to be repeatedly evaluated in the training of neural networks or Bayesian inferences with big data. To reduce the computational cost of such evaluations, the machine learning community are accustomed to Stochastic-Gradient-MCMC methods, which approximate gradients by stochastic ones via uniformly subsampled data points. This, however, introduces extra variance artificially. How to design scalable algorithms that correctly sample the target distribution is an outstanding challenge.

This talk has no ambition of giving a complete solution to this challenge. Instead, it will introduce the problem, and describe our recent heuristic treatment based on second-order Langevin equation coupled with a Metropolis chain. The core of our idea is to replace the uniform subsamplings of training data by non-uniform ones. The objective of this talk is to seek the help from probabilists, so that either better algorithms can be designed or the corresponding stochastic process can be mathematically understood, or both.