ABSTRACT ALGEBRA: EXAM REVIEW

For the final exam, you should know the following definitions:

- Rings and Polynomials
  - (i) Ring
  - (ii) Commutative Ring
  - (iii) Integral Domain
  - (iv) Field
  - (v) Ordered Field
  - (vi) Polynomial Ring
  - (vii) Field Extension
  - (viii) Splitting Field
  - (ix) Ideal
  - (x) Principal Ideal
  - (xi) Principal Ideal Domain
  - (xii) Ring Homomorphism
  - (xiii) Kernel
  - (xiv) Subring
  - (xv) Ring Isomorphism
  - (xvi) Evaluation Homomorphism
  - (xvii) Inclusion Homomorphism
  - (xviii) Product Ring
- Intrinsic Group Theory
  - (i) Group
  - (ii) Abelian Group
  - (iii) Subgroup
  - (iv) Index
  - (v) Normal Subgroup
  - (vi) Order (of an element, of a group)
  - (vii) Conjugacy Class
  - (viii) Coset
  - (ix) Quotient Group
  - (x) Product Group
  - (xi) Group Homomorphism
  - (xii) Kernel
  - (xiii) Group Isomorphism
- Group Actions:
  - (i) Action on a set
  - (ii) Orbit
  - (iii) Stabilizer
  - (iv) Transitive Action
  - (v) Trivial Action
  - (vi) Fixed Point
  - (vii) Permutation Action on Finite Sets

(viii) Symmetry Action on Plane Figures or Platonic Solids
(ix) Linear Action on $\mathbb{R}^n$
(x) Proper Isometry
(xi) Improper Isometry
(xii) Group Actions on Itself: Left Multiplication, Right Multiplication, and Conjugation
(xiii) Group Action on Cosets
(xiv) Group Action on Quotient Group

- Field Theory
  (i) Vector Space
  (ii) Dimension
  (iii) Linear Dependence
  (iv) Span
  (v) Basis
  (vi) Subspace
  (vii) Degree of Field Extension
  (viii) Characteristic
  (ix) Finite Fields $\mathbb{F}_q$
  (x) Symmetry of $\mathbb{F}_q$ over $\mathbb{F}_p$
  (xi) Frobenius Automorphism

You should be able to state and apply the following theorems:

(i) *Fundamental Homomorphism Theorem* for Rings
(ii) *Fundamental Homomorphism Theorem* for Groups
(iii) *Lagrange's Index Theorem* for Groups
(iv) *Orbit-Stabilizer Theorem* for Group actions.
(v) *Classification Theorem for Symmetries of* $\mathbb{R}^3$
(vi) *Classification Theorem for Regular Polyhedra*
(vii) *Burnside's Theorem* for enumerating Orbits
(viii) *Degrees of Field Extensions:* $[K : L][L : F] = [K : F]$
(ix) *Fermat's Little Theorem* for Fields of Finite characteristic
(x) *Existence* and *Uniqueness* of Finite Fields $\mathbb{F}_q$

The exam will have four parts. One will deal with ring homomorphisms, ideals and quotient rings, the second will involve conjugacy classes and subgroups of a linear group or a permutation group, the third will ask you to count orbits of a group action on a geometric figure, and the fourth will be field theory.