

Products and Permutations

1. PROPERTIES OF THE PRODUCT FUNCTOR

In each of the following, let A , B , G and H be groups.

- a) Prove that $G \times H$ is isomorphic to $H \times G$.
- b) Given a homomorphism $f: A \rightarrow G$, construct a homomorphism from $A \times H$ to $G \times H$. Find its kernel and image.
- c) Prove that $A \times H$ is isomorphic to $G \times H$ if and only if A is isomorphic to G .
- d) Given homomorphisms $f: A \rightarrow G$, $g: B \rightarrow H$, construct a homomorphism from $A \times B$ to $G \times H$. Find its kernel and image.
- e) Given homomorphisms $f: A \rightarrow G$, $g: A \rightarrow H$, construct a homomorphism from A to $G \times H$. Find its kernel and image.

2. PERMUTATIONS

- a) In S_9 , write the following in function notation: $p = (12)(45)(78)$,
 $q = (135)(79)$, $r = (2468)$, pq , qr , pr .
- b) Write out the elements of S_3 in function and cycle notations.
- c) What do you obtain when you multiply two 2-cycles in S_3 ? Two 3-cycles? A 2-cycle by a 3-cycle?