APMA 1650 - Spring 2021

Midterm Exam 1

Please read the following instructions carefully:

- There are 5 actual problems on this exam. You must show all work and explain/justify your answers. Answers without sufficient justification may not receive full credit. Partial credit for wrong answers may be given if you show your thought process.
- The exam is open book. This means you may use any resources available in the textbook, Canvas or the course webpage. If you use any of these sources, you must indicate when and where you use them. No other sources are allowed.
- No communication with anyone other than course staff about the exam or course material is allowed during the exam window from 11:00am EST 2/22 to 2:00pm EST 2/23.
- Questions during the exam should be posted **privately** to course staff on Campuswire. Do NOT post or answer public questions during the 24hr exam period.
- Calculators are allowed. You can use any of the combinatorial and CDF calculators located in the online textbook. You can also leave answers as basic fractions, elementary functions or known combinatorial quantities unless otherwise indicated.
- Please write each problem on a **separate sheet of paper**. Make sure to use large dark lettering and good light when scanning before uploading to Gradescope, just like you would for homework.
- Give yourself at least 20 minutes to upload your answers and check your work. DO NOT race to upload everything at the last minute!
- Technical problems must be communicated **immediately**, via whatever you can get to work.
- Take a deep breath before you begin

Problem 0. (Playing by the rules) (0 pts) You must read and rewrite the following statements for this exam to be graded

- I have read and understood the above instructions. I understand that failure to follow these instructions may result in loss of points or no credit.
- I have not/nor do I plan to communicate or share information about the exam or course material with anyone besides course staff during the exam window.
- I have not used/nor plan to use any unapproved sources during the exam window.
- I understand that violating these rules may result in referral to the honor committee.



Figure 1: A $4\times 3\times 3$ Rubik's cube

Problem 1. (Rubik's rectangular prism) (20pts) Suppose you have a $4 \times 3 \times 3$ Rubik's "cube" that has 4 sides that are 4×3 , and 2 sides that are 3×3 (see Figure 1). Every side of the puzzle has a unique color to start with. If you're allowed to remove all the stickers and replace them in any position, how many unique ways are there to color the "cube" assuming each side is distinct (i.e. you have a way of uniquely identifying each side that fixes an orientation)?

Problem 2. (Campuswire) (20 pts) Campuswire isn't loading and you are running out of patience. Every time you try to load the page, it has a probability of 1/4 of actually loading. You wake up this morning full of patience and decide to try to load Campuswire. Let X be the number of tries it takes to load.

- a. (10 pts) What is the probability that X equals an odd number $1, 3, 5, \ldots$? Write your answer as a fraction in simplest form.
- b. (10 pts) Every time a page fails to load you lose patience. Lets assume your patience level starts this morning at the value a and after every failed page-load it gets halved (so after one failed page load you have a/2 patience left and after two failed attempts you have a/4). What is your expected patience level when the page finally loads? Write your answer as a fraction in simplest form.

Problem 3. (Recruiters) (20 pts) A job recruiter is looking to hire someone for a fancy data science job. They have two lists: List 1 contains the names of 5 APMA students and 2 CSCI students. List 2 contains the names of 2 APMA students and 6 CSCI students. One of the names is then randomly selected from list 1 and added to list 2. A final name is then randomly selected from the newly augmented list 2. Given that the final name selected was a CSCI student, what is the probability that an APMA student's name was originally selected from list 1 and added to list 2?

Problem 4. (Conflicts) (20 pts) Suppose m professors randomly choose from n time slots to hold their final exams. If two professors pick the same time slot, we say that they are in conflict. (If three professors all pick the same time slot, that gives three pairs of professors in conflict.) What is the expected number of pairs of professors in conflict? Your answer should depend on m and n.

Problem 5. (Powerball) (20 pts) Suppose in a lottery you pick five different numbers from 1 to 90. Then five different winning numbers are drawn at random from 1 to 90.

- a. (10 pts) Let X be the number of winning numbers that you picked (regardless of order). What is the range and PMF of X?
- b. (10 pts) If you pick two winning numbers, you win 20 dollars. For three, you win 150 dollars. For four, you win 5,000 dollars. If all five match, you win a million dollars! What are your expected winnings? Give an exact answer to 3 decimal places. (You may use a combinatorial calculator). For partial credit, give a formula that involves the PMF from part a).