**Problem 1: Potpourri** (20 pts)

(a) (4 pts) Let $X$ be a discrete random variable with mean $\mu = E(X)$. Show the following formula

$$E(X-\mu)^2 = E(X^2) - \mu^2.$$

Find $E(X)$, and $\text{Var}(X)$.

(b) (8 pts) Let $X$ be a discrete random variable with mean $\mu = E(X)$ and variance $\sigma^2 = \text{Var(X)}$. Show the following formula

$$E(X-\mu)^3 = E(X^3) - 3\mu\sigma^2 - \mu^3.$$

(c) Not covered (8 pts) Suppose $X_1, X_2, X_3 \ldots, X_n$ are independent random variables with a common mean $E(X_i) = \mu$ and variance $\text{Var}(X_i) = \sigma^2 > 0$. Let

$$\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$$

be the sample mean. Show that

$$P(|\overline{X}_n - \mu| > \sigma) \leq \frac{1}{n}.$$

**Problem 1** *(Extra page)*

**Problem 2: One fish two fish** (18 pts)
Suppose you work at a pet store and have five identical fish bowls. Inside each bowl are 5 fish, comprised of a mix of red and blue fish. The bowls are labeled $1, 2, 3, 4, 5$ indicating the number of red fish in each bowl (e.g. bowl 2 has exactly two red fish). A customer wants two red fish. Since you are in a rush, you pick a bowl at random and pick out two fish from that bowl (also at random).

(a) (8pts) What is the probability that both fish you selected are red?

(b) (10pts) You are in luck! Both fish are red. What is the probability that you selected bowl 3?

**Problem 2** *(Extra page)*

**Problem 3: The mannersbit** (25 pts)
Suppose a donut is left out at the office in the morning for anyone who wants to eat it. Being too polite to eat everything that's left out, each person who comes across it eats only half of what's left, leaving the remaining half for the next person. Suppose that number people who come across the donut in a given work day is Poisson distributed with an average rate of 10 people per work day.

(a) (8 pts) Assuming an 8 hour work day, what is the probability of exactly one person coming by the donut in a given hour?

(b) (5 pts) If exactly $n$ people come across the donut in a given work day (all at different times) what fraction of the donut is left at the end of the work day?

(c) (12pts) What is the expected fraction of the donut left over at the end of a given work day? (For full credit, give a simple answer that is not an infinite series)

**Problem 3** *(Extra page)*

**Problem 4: Exams of the round table** (25 pts)
Suppose there are 20 students seated at a round table to take an exam. The professor has made 4 different exams (5 of each type) and distributes them randomly among the students (so that each arrangement is equally likely).

(a) (5 pts) How many unique ways are there to distribute the different exam types among the students?

(b) (8 pts) What is the probability that a given student is seated next to someone with the same exam type?

(c) (12 pts) What is the expected number of students who are seated next to someone with the same exam type?

**Problem 4** *(Extra page)*

**Problem 5: Odd one out** (12 pts)
A group of 16 students (4 ECON and 12 APMA) are randomly divided into 4 study groups of size 4.

1. (4 pts) How many ways are there to divide all the students into the four study groups?

2. (8 pts) What is the probability that each study group includes an ECON student?