

APMA 1650
CHE

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1) Let X be the number of the cars being repaired at a repair shop. We have the following information:

At any time, there are at most 3 cars being repaired.

The probability of having 2 cars at the shop is the same as the probability of having 1 car.

The probability of having no car at the shop is the same as the probability of having 3 cars.

The probability of having 1 or 2 cars is half of the probability of having 0 or 3 cars.

Find the PMF of X

2) The number of emails that I get in a weekday (Monday through Friday) can be modeled by a Poisson distribution with an average of $(1/6)$ emails per minute. The number of emails that I receive on weekends (Saturday and Sunday) can be modeled by a Poisson distribution with an average of $(1/30)$ emails per minute

a) What is the probability that I get no emails in an interval of 4 hours on a Sunday?

b) A random day is chosen (all days of the week are equally likely to be selected), and a random interval of 1 hour is selected on the chosen day. It is observed that I didn't receive any emails in that interval. What is the probability that the chosen day is a weekday?

3) Let X be a discrete random variable with the following pmf:

$$P_X(k) = \begin{cases} 0.5 & \text{for } k = 1 \\ 0.3 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

a) Find $E[X]$

b) Find $\text{Var}[X]$ and $\text{SD}[X]$

c) If $Y = 2/X$, find $E[Y]$

4) The SciLi has 14 floors numbered 1 to 14. 20 people get on the elevator together at the floor A (the lowest floor of the building), and each gets off at one of the 14 floors, uniformly at random (and independently of everybody else). What is the expected number of floors the elevator stops at? Use linearity of expectations.

5) A group of six software packages available has been ranked from 1 to 6 (best to worst). An engineering firm, unaware of the rankings, randomly selected and then purchased two of the packages. Let X denote the number of packages purchased by the firm that are ranked 3, 4, 5, or 6. Give the probability distribution for X .

6)

a) Let $X \sim \text{Geo}(p)$. Find $P(X = \text{an even number})$.

b) Suppose we have a bag which contains 7 green balls and 23 yellow balls. We draw a ball randomly. We check the color of the ball. After checking the color, we put the ball back to the bag.

i) What is the probability that the number of the draw on which the first yellow ball is obtained is at least 5?

ii) What is the expected number of the draw on which the first yellow ball is obtained?

