

# APMA 1650 - Spring 2021

## Midterm Exam 2

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Please read the following instructions carefully:

- There are 5 actual problems on this exam. **You must show all work and explain/justify your answers.** Answers without sufficient justification may not receive full credit. Partial credit for wrong answers may be given if you show your thought process.
- **The exam is open book.** This means you may use any resources available in the textbook, Canvas or the course webpage. If you use any of these sources, you must indicate when and where you use them. **No other sources are allowed.**
- **No communication with anyone other than course staff** about the exam or course material is allowed during the exam window from 11:00am EDT 3/31 to 3:00pm EDT 4/01.
- Questions during the exam should be posted **privately** to course staff on Campuswire. Do NOT post or answer public questions during the 24hr exam period.
- **Calculators are allowed.** You can use any of the combinatorial and CDF calculators located in the online textbook. You can also leave answers as basic fractions, elementary functions or known combinatorial quantities unless otherwise indicated.
- Please write each problem on a **separate sheet of paper**. Make sure to use large dark lettering and good light when scanning before uploading to Gradescope, just like you would for homework.
- Give yourself **at least 20 minutes** to upload your answers and check your work. DO NOT race to upload everything at the last minute!
- Technical problems must be communicated **immediately**, via whatever you can get to work. I can upload a submission for you if Gradescope is giving you problems.
- **Take a deep breath before you begin**

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**Problem 1. (Playing by the rules)** (0 pts) You must read and rewrite the following statements for this exam to be graded

- *I have read and understood the above instructions. I understand that failure to follow these instructions may result in loss of points or no credit.*
- *I have not/nor do I plan to communicate or share information about the exam or course material with anyone besides course staff during the exam window.*
- *I have not used/nor plan to use any unapproved sources during the exam window.*
- *I understand that violating these rules may result in referral to the honor committee.*

**Problem 2. (Ever Given)** (20pts) In a given year, the proportion of time  $T$  that a particular canal is blocked has a probability density function

$$f_T(t) = \begin{cases} 2(1-t), & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

The global economic cost of this blockage (in billions of dollars) is given by

$$C = 10T + 4T^2.$$

- a. (4 pts) Find the mean of  $C$ .
- b. (8 pts) Find the variance of  $C$ .
- c. (8 pts) Use Chebyshev to find a value  $b$  such  $P(C \leq b)$  is at least 0.75. (State your answer to two decimal places)

**Problem 3. (Quality Balls)** (20 pts) A quality control process for a basketball manufacturing plant samples 10 finished basketballs a day and counts  $X$  the number of defective basketballs in that sample. The probability  $p$  that a given basketball is defective varies from day to day and is assumed to have a uniform distribution on  $[0, 1/4]$ .

- a. (6 pts) For a given  $p$ , what is the conditional distribution of  $X$ ? (You can give a name and appropriate parameters)
- b. (6 pts) Find  $EX$ .
- c. (8 pts) Find  $\text{Var}(X)$ .

**Problem 4. (Joint Ratio)** (20 pts) Suppose that  $X$  and  $Y$  are jointly continuous random variables with joint distribution

$$f_{XY}(x, y) = \begin{cases} \frac{c}{x} & 0 < x \leq 1, 0 \leq y \leq 2x \leq 2 \\ 0 & \text{otherwise} \end{cases}.$$

- a. (6 pts) Find the value  $c$  that makes  $f_{XY}(x, y)$  a well defined joint density.
- b. (6 pts) What are the marginals  $f_X(x), f_Y(y)$ ?
- c. (8 pts) Let  $R = Y/X$ . Find the CDF and PDF for  $R$ . Be sure to specify its range. (Hint: write  $P(R \leq r)$  as an area integral over a certain region)

**Problem 5. (Standard Normality)** (20 pts) Let  $X \sim N(\mu_X, \sigma_X^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2)$  be two independent normal random variables and let  $Z = X + aY + b$ , for two numbers  $a, b \in \mathbb{R}$ .

- (6 pts) Find the mean and variance of  $Z$ .
- (8 pts) Find the correlations,  $\rho(Z, X)$  and  $\rho(Z, Y)$ .
- (6 pts) Suppose that  $\sigma_X^2 \leq 1$ . Using the fact that the sum of two normal random variables is again normal, find the values of  $a$  and  $b$  so that  $Z \sim N(0, 1)$ . Your answer should depend only on  $\mu_X, \mu_Y, \sigma_X$  and  $\sigma_Y$ .

**Problem 6. (Mix it up)** (20 pts) Suppose a random variable  $Y$  has a CDF given by

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \leq y < 1/2 \\ y & 1/2 \leq y < 1 \\ 1 & y \geq 1. \end{cases}$$

- (6 pts) Write  $F_Y(y) = C(y) + D(y)$ , where  $C(y)$  is the continuous part and  $D(y)$  is the discrete part (a staircase function).
- (8 pts) Find the generalized PDF  $f_Y(y)$ .
- (6 pts) What is the expected value of  $Y$ ?