APMA 1650

Easy Practice Midterm Exam 2

Problem 1. (Conditional) Let X and Y be random variables with joint density given by:

$$f_{XY}(x, y) = \begin{cases} cx & \text{if} \quad 0 \le x \le 1 \quad \text{and} \quad 0 \le x^2 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of c for which $f_{XY}(x, y)$ is a valid density.

b. Find $E[X]$.

c. Find the conditional density of X given $Y = y$.

Problem 2. (Covariance) Let X be the score of a random student on a final exam and let Y be the number of hours spent studying. Suppose that X and Y are related by $X = 2Y + Z$, where Z is independent of Y. Further suppose $\text{Var}(X) = 40$ and $\text{Var}(Y) = 2$.

a. What is $\text{Var}(Z)$?

b. What is $\text{Cov}(X, Y)$?

Problem 3. (Alice and Bob) Alice and Bob each uniformly and independently select a point from the interval $[0, 2]$.

a. What is the joint distribution of the two chosen points?

b. What is the probability that the distance between these two points is no more than 1?

Problem 4. (Stick) Suppose a stick of length 1 is broken in two places. The first break point is chosen uniformly at random along the length of the stick from $[0, 1]$. The second break point is chosen uniformly at random from 0 to the first break point.

a. Find the joint probability distribution of the two break points. (Be careful about your bounds.)

b. What is the covariance of the two break points?

Problem 5. (Bounds) Let X be a binomial random variable,

$$f_X(x) = \begin{cases} e^{x+1} & x \le -1 \\ 0 & \text{otherwise} \end{cases}$$

a. Give a lower bound using Chebyshev for $P(-4 \le X \le 0)$.

b. Use Chebyshev to determine an a such that $P(X \ge a) \ge 0.95$