APMA 1650 Final Exam Checklist

The exam will be broken down into 50% on new material (limit theorems, estimators and confidence intervals) 25% on material from midterm 1 and 25% material from midterm 2. Some fundamental concepts like random variables, expectation, conditional probability, etc are cumulative and permeate everything we have done in the class.

Stuff from Midterm 1 (25% of the exam)

1. Basics of set theory: Make sure you understand how to manipulate sets and the various operations that can be performed.
   (a) Union, intersection, complement, mutually exclusive, Venn diagrams
   (b) DeMorgans Law and algebraic properties of sets
2. Fundamentals of probability
   (a) laws of addition, complement, difference
   (b) how to calculate probabilities by splitting up events and adding and subtracting their probabilities
   (c) Venn diagrams
3. Discrete probability and combinatorics
   (a) Equally likely events, counting, rule of products, permutations, combinations, partitions
   (b) Sampling with/without replacement un/ordered
   (c) Make sure you understand the formulas for permutations, combinations and partitions
4. Conditional probability
   (a) Know the definition and what it means intuitively
   (b) Law of multiplication, law of total probability, probability trees
   (c) Independence, what does it mean, how to check it
   (d) Bayes rule, and inverting conditional probabilities
5. Discrete random variables and distributions
   (a) PMF and CDF
   (b) Expected value and variance. Know their properties and how use use them (eg: linearity and how variance rescales)
   (c) Special distributions: Bernoulli, Binomial, Geometric, Poisson, Pascal. Know their properties, what they describe and how they relate.
   (d) What happens to a PMF of an RV when you apply a function to the RV?

Stuff from Midterm 2 (25% of the exam)

1. Continuous random variables and distributions
   (a) Probability density function, cumulative distribution functions
   (b) Expected value and variance again
   (c) Special Distributions: Uniform distribution, exponential distribution (how it relates to geometric and Poisson), normal distribution, Gamma distribution (how it relates to exponential and Pascal).
   (d) CDF method. What is the distribution of $h(X)$?
2. Multivariate distributions
   (a) Joint distributions and CDFs, iterated integrals and integrating over regions
   (b) Marginals, conditional distributions, independent random variables
   (c) Covariance and correlation. Relation to independence. Can two random variables have no correlation, but be dependent? Do you have an example?
   (d) Conditional expectation and conditional variance. How to use them to compute unconditional variance and expectation (law of total expectation law of total variance)

New material (50% of the exam)

1. WLLN, CLT and convergence of random variables
   (a) What is the weak law of large numbers and what does it say?
   (b) Chebyshev's inequality, what does it mean and how to apply it to bound probabilities.
   (c) How to prove the weak law of large numbers from Chebyshev's inequality.
   (d) What is the central limit theorem? Why is it so important? How do you apply it?
   (e) Compare CLT to WLLN.
   (f) What is convergence in probability and distribution? Which one is stronger? How do they relate to WLLN and CLT? Does convergence in probability and distribution still hold if you apply a continuous function? What about if you multiply two random variables converging in distribution?
2. Sampling and estimation
   (a) iid random samples what are they?
   (b) Point estimators, what are they? What are some examples?
   (c) Bias of an estimator. How to make a biased estimator unbiased.
   (d) Mean square error (MSE), it's relation to variance and bias of an estimator, know how to compute.
   (e) How to show that one estimator is better than another using MSE.
3. Consistency

(a) What does it mean to show an estimator is consistent. How to use the MSE to show consistency.

(b) How to use the weak law of large numbers to show consistency. How does this relate to MSE?

(c) Properties of convergence in probability and how to use it to show consistency.

4. Methods of estimation

   (a) Maximum likelihood estimators (MLE). How to compute them. How are they related to sufficient statistics?

   (b) Why taking the logarithm makes things easier (also very useful in numerics and semi-definite programing for the computer science folks)

   (c) Properties of MLEs? Consistency, asymptoticly unbiased, CLT.

5. Interval estimators

   (a) What is a $(1-\alpha)$ confidence interval? Why is it useful?

   (b) Pivotal quantities. What are they? How to use them to calculate confidence intervals.

   (c) How to use the CLT to get confidence intervals of means if the sample size is large.

   (d) $\chi^2(n)$ distribution. What does it describe? What are it's properties? What is $\chi^2_{p,n}$? How do you find it's value?

   (e) How to find $(1-\alpha)$ confidence intervals for the variance in normal samples using $\chi^2(n)$.

   (f) t-distribution. Why is it useful? What are it's properties? What is $t_{p,n}$? How do you find it's value?

   (g) How to use the t-distribution to calculate $1-\alpha$ confidence intervals if the sample population is normal (and the sample size is not large)