APMA 1650 - Calculus Preparedness Assignment

Instructions:

The following practice assignment is to check for your calculus preparedness. It is meant to give you a taste of the type of calculus problems that you are expected to complete. If you find some of the problems challenging, please review your calculus basics.

It is also meant to allow you to practice submitting your homework assignments in Gradescope with no repercussions. The TAs will grade each problem and give feedback based on your work and let you know where you can improve. **This assignment will not count towards your final grade**.

As with all of the homework assignments, for "full credit" please show **all work** and **explain your answers** in as much detail as possible. Simply stating the answer will earn you zero credit.

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1. Let $f(x)$ be a continuous function on $\mathbb{R} = (-\infty, \infty)$ and let

$$F(x) = \int_{-\infty}^{x} f(y)\,\mathrm{d}y.$$

What is $F'(x)$? Be sure to name any fundamental theorems you use.

2. (Derivatives)

(a) Calculate

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln(x)$$

(b) Calculate

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln(\ln(x))$$

(c) Let $p \geq 0$. Calculate

$$\lim_{x \to 0} x^{1+p} \ln(x)$$

3. (Series)

(a) Let $0 \leq \lambda < \infty$. Calculate

$$\sum_{k=1}^{\infty} \frac{\lambda^k 2^k}{k!}$$

(b) Calculate a simple formula for

$$\sum_{k=0}^{n} e^{-\lambda k}.$$

(c) Use the answer to part (b) and the fact that $\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\sum_{k=0}^{n} e^{-\lambda k}\right) = -\sum_{k=0}^{n} k e^{-\lambda k}$ to calculate

$$\sum_{k=1}^{n} k.$$

(d) Use the answer to part (b) and the fact that $\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\left(\sum_{k=0}^{n} e^{-\lambda k}\right) = \sum_{k=0}^{n} k^2 e^{-\lambda k}$ to calculate

$$\sum_{k=1}^{n} k(k-1).$$

4. (Integrals)

(a) CAlculate

$$\int_0^1 (1-x^2)^2 \mathrm{d}x$$

(b) Let $-\infty < \theta < \infty$, calculate

$$\int_{-\infty}^{\infty} \frac{1}{2} x e^{-|x-\theta|} \mathrm{d}x.$$

(Hint: use symmetry)

(c) Calculate

$$\int \ln(z)\,\mathrm{d}z$$

(d) Let $p \geq 0$. Calculate

$$\int_0^1 x^p \ln(x)\,\mathrm{d}x.$$