APMA 1650

Homework 2

Instructions: Homework is due by 11:59pm EST in Gradescope on the day listed on the course webpage. You must *show all work* to get full credit. You can use calculators for this assignment. Solutions must be written independently and *cannot* be shared with any other students. *There will be a 5pt penalty for homework submitted with problems incorrectly assigned to a page.*

---

1. (14 pts) Let A, B, C be three events with $P(A), P(B), P(C) \neq 0$ and suppose that $P(A|B) \neq 0$ and $P(A|C) \neq 0$. If B and C are mutually exclusive, show that the following formula always holds

$$P(A|B \cup C) = P(A|B)\left(\frac{P(B)}{P(B)+P(C)}\right) + P(A|C)\left(\frac{P(C)}{P(B)+P(C)}\right).$$

Use this to conclude that the following formula **cannot hold**

$$P(A|B \cup C) = P(A|B) + P(A|C).$$

Namely probabilities are **not** additive in their conditional sets. (Hint: Mathematically speaking, it is sufficient to assume that the above formula holds and then deduce something that contradicts the assumptions. This is called a 'proof by contradiction')

2. (18 pts) Suppose you flip a fair coin twice (assume that each flip is independent). Let A be the event that the first flip is heads, B be the event that the second flip is tails, and C be the event that both flips give the same outcome.

a. (4 pts) Are A and B independent?

b. (4 pts) Are B and C independent?

c. (4 pts) Are C and A independent?

d. (6 pts) Are A, B and C independent?

3. (22 pts) Suppose you have a collection of 6 bags, each containing a total of 5 marbles (a mixture of red marbles and green marbles). The bags are labeled 0,1,2,3,4,5 indicating the number of red marbles in each bag. You pick a bag at random and pull out two marbles.

a. (10 pts) What is the probability that both marbles you selected are red? Draw a probability tree.

b. (12 pts) Suppose both marbles are red. What is the probability that you selected bag 2?

4. (28 pts) In your area it is estimated that 1 in 50 people are currently infected with COVID. The BinaxNOW COVID antigen test has been shown to have a false positive rate of 1% and a false negative rate of 16%.

a. (8 pts) You want to see your family this weekend, but want to be 90% sure you don't have COVID. Supose you receive a negative antigen test, what is the probability that you don't have COVID? Should you go see your family?

b. (8 pts) Suppose you receive a positive antigen test. What is the probability that you actually have COVID?

c. (12 pts) The local government sets new regulations requiring that the percentage of positive COVID test results that are true positives, known as positive predictive value (PPV), be at least 90% (assuming that the 1 in 50 COVID prevalence remains fixed). Abbot (the pharmaceutical company that makes the test) has two options: they can invest resources in decreasing the false positive rate, or they can invest resources in decreasing the false negative rate. Which one should they do? How small should they set the rate to comply with the new regulations.

5. (18 pts) You are on a jury for a robbery trial. The suspect is accused of stealing a cookie from a cookie jar. The prosecution has just presented evidence that the suspect was seen eyeing the cookie jar before the cookie was stolen and therefore probably stole the cookie. The defense then presents a statistic that only 1 in 1000 people eyeing cookie jars actually end up stealing a cookie from it.

a. (4 pts) Assuming this 1 in 1000 statistic is correct. Should we conclude that there is only a 1/1000 probability the suspect actually stole the cookie? Briefly explain in words.

b. (10 pts) Somehow you were able to ascertain that among cookie jars that were recently eyed by someone, only 0.2% had a cookie stolen by someone other than the person eyeing them. Suppose you randomly select a cookie jar that was just "eyed" by someone. Calculate the probability the person eyeing the cookies ends up stealing the cookie, given that a cookie was indeed stolen by someone.

c. (4 pts) Based on you calculation in part (b), do you agree or disagree with the argument made by the defense? Do you consider it relevant that the suspect was eyeing the cookie jar before the cookie was stolen? Would you judge it more accurate to use 1/1000 or the number you calculated in part (b) as the likelihood that the suspect stole the cookie. Why?