APMA 1650

Homework 4

Instructions: Homework is due by 11:59pm EST in Gradescope on the day listed on the course webpage. You can use calculators for this assignment. Solutions must be written independently and *cannot* be shared with any other students.

You must *show all work* and *explain your answers thoroughly* to get full credit. **You will be graded partly on how well you explain the answers**.

There will be a 5pt penalty for homework submitted with problems incorrectly assigned to a page. A 10pt penalty will be applied for homework submitted during the late window.

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1. (18 pts) Consider the following CDF of a random variable X

$$F_X(x) = \begin{cases} 0 & \text{for } x < -2 \\ 1/3 & \text{for } -2 \leq x < -1 \\ 1/2 & \text{for } -1 \leq x < 1 \\ 5/6 & \text{for } 1 \leq x < 2 \\ 1 & \text{for } 2 \leq x \end{cases} .$$

Find the following

a. (4 pts) $P(X > -2)$

b. (4 pts) $P(-1 \leq X \leq 2)$

c. (4 pts) $P(2X \leq -1)$

d. (6 ots) $E[X]$

2. (20 pts) A TV manufacturer makes a display that has standard 1920×1080 pixel array. Suppose that each pixel independently has a one in a million chance of being defective. Use the Poisson distribution to answer the following problems. (Hint: you may find the textbook Poisson CDF calculator very useful).

a. (10 pts) In order for a display to meet the ISO 9241-305 Class II standard, the manufacturer can't have more than 2 defective pixels per one million pixels in that display. Approximate the probability that a given TV does not meet the Class II standard.

b. (10 pts) Now suppose that the manufacturer needs to make 100 TVs that meet the Class II standard. Approximate the minimal number of TVs they should manufacture to be 99% certain they have at least 100 Class II TVs produced. (Hint: make an educated guess based on the mean and variance of a certain Poisson random variable and use the CDF calculator to "guess and check" until you find the right value.)

3. (18 pts)

a. (6 pts) Let $X \sim \text{Geometric}(p)$. Calculate $\text{Var}(X)$.

b. (6 pts) Let $Y \sim \text{Pascal}(m, p)$. Calculate $\text{Var}(Y)$.

c. (6 pts) Let $X \sim \text{Poisson}(\lambda)$. Calculate $\text{Var}(X)$.

4. (14 pts)

a. (4 pts) Suppose X is a random variable with $E[X] = 1$ and $E[X(X-2)] = 3$. What is $\text{Var}(X)$?

b. (4 pts) Suppose X is a random variable with mean 1. Show that $E[X^2] \geq 1$.

c. (6 pts) Let $X = I_A$ be the indicator Bernoulli random variable for a probability zero event A. What is $\text{Var}(X)$?

5. (30 pts) Suppose that for a (very easy) homework assignment students are tasked with flipping a fair coin 100 times and recording the outcomes. One student does the work and writes down the results of the 100 flips, another student is lazy and makes up the data. Here are the outcomes from the two students (in no particular order):

Student 1:

$THHHTHTTTTHTTHTTTHHTHTTHT$

$HHHTHTHHTHTTHHTTTTHTTTHTH$

$TTHHTTTTTTTTHTHHHHHTHTHTH$

$THTHTHHHHHTHHTTTTTHTTHHTH$

Student 2:

$HTTHTTHTHHTTHTHTHTTHHTHTT$

$HTTHHHTTHTTHTHTHTHHTTHTTH$

$THTHTHTHHHTTHTHTHTHHTHTTT$

$HTHHTHTHTHTHHTTHTHTHTTHHT$

Your goal is to figure out which student was most likely to have fudged the data. Your strategy is to count the number of TT pairs in each sample. (Here counting TT pairs involves counting how many times TT shows up out of *all* 99 *neighboring* pairs. For instance for the 8 tosses $TTTHTHTT$ has 3 TT pairs.)

a. (12 pts) Suppose you flip a fair coin n times. What is the expected number of TT pairs? (Hint: Use sums of Bernoulli random variables to count the number of occurrences.)

b. (3 pts) Based on your answers to a, which student do you think most likely fudged their data.

c. (15 pts) Repeat a and b for the number of TTT triples (i.e. $TTTTT$ has 3 triples).