

Tropical Algebra

Exercises for the Clifford Lectures
at Tulane University, November 12-15, 2008

1. Install Anders Jensen's software **GFan** on your computer. Read *Section 4: Doing tropical computations* of the manual and try an example.
2. The discriminant of a quadratic form in three unknowns equals

$$\frac{1}{2} \det \begin{pmatrix} 2a & b & c \\ b & 2d & e \\ c & e & 2f \end{pmatrix} = 4adf + bce - ae^2 - dc^2 - fb^2.$$

Draw a picture of the tropical hypersurface defined by this polynomial.

3. Pick three linear forms in six variables and let I be the ideal they generate. Compute the tropical linear space $\mathcal{T}(I)$. Draw a picture.
4. Determine the image of the linear map $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by the matrix

$$\begin{pmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

Compute the determinant, the eigenvalue and eigenspace of this matrix.

5. In the lecture, we drew the tropical hypersurface of the 3×3 -determinant. Pick one point each from a triangle and a quadrangle, and find two singular matrices $\mathbb{C}\{\{t\}\}^{3 \times 3}$ whose valuations are your two matrices.
6. Compute and draw the tropical variety defined by the 3×3 -minors of

$$\begin{pmatrix} a & b & c & d \\ b & c & d & e \\ c & d & e & f \\ d & e & f & g \end{pmatrix}$$