

**MATH 122 - CALCULUS II**  
PRACTICE FINAL EXAM

**Problem 1.** The area enclosed by the parabola  $y^2 = 2x + 6$  and the line  $y = x - 1$  is

- (a) 14    (b) 16    (c) 18    (d) 20.

**Problem 2.** The volume of the solid obtained by rotating the region bounded by  $y^2 = x$  and  $x = 2y$  about the  $y$ -axis is

- (a)  $\frac{4\pi}{3}$     (b)  $\frac{8\pi}{3}$     (c)  $\frac{16\pi}{5}$     (d)  $\frac{64\pi}{5}$ .

**Problem 3.** A tank in the shape of the bottom half of a sphere of radius 1 meter is filled with water. How much work does it take to pump all of the water out of the top of the tank?

- (a)  $\frac{343\pi}{20} J$     (b)  $\frac{7\pi}{4} J$     (c)  $17,150\pi J$     (d)  $2,450\pi J$ .

**Problem 4.** Which of the following improper integrals is convergent?

- (a)  $\int_1^\infty \sin x dx$     (b)  $\int_1^\infty \frac{x-1}{\sqrt{x^2+1}} dx$     (c)  $\int_1^\infty \frac{x}{1+e^x} dx$     (d)  $\int_1^\infty \frac{2+\cos x}{x} dx$ .

**Problem 5.** The integral  $\int \frac{dx}{x^2-9}$  is

- (a)  $\frac{1}{6} \ln \left| \frac{x-3}{x+3} \right| + C$     (b)  $\frac{1}{6} \ln \left| \frac{x+3}{x-3} \right| + C$     (c)  $6 \ln \left| \frac{x-3}{x+3} \right| + C$     (d)  $6 \ln \left| \frac{x+3}{x-3} \right| + C$ .

**Problem 6.** The integral  $\int \frac{1}{1+4x^2} dx$  is

- (a)  $\tan^{-1}(4x) + C$     (b)  $\tan^{-1}(2x) + C$     (c)  $\tan^{-1}\left(\frac{x}{2}\right) + C$     (d)  $-\frac{1}{1+4x} + C$ .

**Problem 7.** The integral  $\int \sin^5 \theta \cos^2 \theta d\theta$  is

- (1)  $-\frac{\cos^3 \theta}{3} + \frac{2 \cos^5 \theta}{5} - \frac{\cos^7 \theta}{7} + C$   
 (2)  $\frac{\cos^3 \theta}{3} + \frac{2 \cos^5 \theta}{5} - \frac{\cos^7 \theta}{7} + C$   
 (3)  $\frac{\cos^3 \theta}{3} + \frac{2 \cos^5 \theta}{5} + \frac{\cos^7 \theta}{7} + C$   
 (4)  $-\frac{\cos^3 \theta}{3} - \frac{2 \cos^5 \theta}{5} - \frac{\cos^7 \theta}{7} + C$ .

**Problem 8.** The integral  $\int \tan^6 \theta \sec^4 \theta d\theta$  is

- (1)  $\frac{\tan^7 \theta}{7} - \frac{\tan^9 \theta}{9} + C$
- (2)  $\frac{\tan^7 \theta}{7} + \frac{\tan^9 \theta}{9} + C$
- (3)  $\frac{\sec^7 \theta}{7} - \frac{\sec^9 \theta}{9} + C$
- (4)  $\frac{\sec^7 \theta}{7} + \frac{\sec^9 \theta}{9} + C.$

**Problem 9.** The arc length of the curve  $y = \frac{2}{3}(x-1)^{\frac{3}{2}}$  from  $x = 1$  to  $x = 4$  is

- (a)  $\frac{16}{3}$
- (b)  $\frac{14}{3}$
- (c)  $\frac{21}{2}$
- (d) 12.

**Problem 10.** The surface of revolution formed by revolving about the  $x$ -axis the curve  $y = 2\sqrt{x}$  from  $x = 3$  to  $x = 8$  is

- (a)  $\frac{152\pi}{3}$
- (b)  $\frac{40\pi}{3}$
- (c)  $\frac{8\pi}{3}[8\sqrt{8} - 3\sqrt{3}]$
- (d)  $\frac{168\pi}{3}.$

**Problem 11.** Given the initial-valued problem  $y' = e^x + y$  and  $y(0) = 1$ . Approximating  $y(1)$ , using Euler's method with step size  $h = 0.5$  gives

- (a)  $y(1) \approx \frac{3}{2}$
- (b)  $y(1) \approx 2$
- (c)  $y(1) \approx 2 + \frac{\sqrt{e}}{2}$
- (d)  $y(1) \approx 3 + \frac{\sqrt{e}}{2}.$

**Problem 12.** The solution to the differential equation  $y' = \frac{x^2}{y^2}$  is

- (1)  $y = \sqrt[3]{-x^3 + C}$
- (2)  $y = \sqrt[3]{x^3 + C}$
- (3)  $y = \sqrt{-x^3 + C}$
- (4)  $y = \sqrt{x^3 + C}.$

**Problem 13.** Given the initial-valued problem  $\begin{cases} y' + 2xy = e^{-x^2} \\ y(0) = 1 \end{cases}$ . Then  $y(1)$  is

- (a)  $\frac{2}{e}$
- (b) 2
- (c) 0
- (d)  $\frac{1}{e}.$

**Problem 14.** Let  $C$  be a parametric curve given by  $\begin{cases} x = t^2 \\ y = t^3 - 3t \end{cases}$ . The number of tangent lines at  $(3,0)$  is

- (a) 1
- (b) 2
- (c) 3
- (d) 4.

**Problem 15.** The area under one arch of the cycloid  $\begin{cases} x = r(\theta - \sin \theta) \\ y = r(1 - \cos \theta) \end{cases}$  is

(a)  $3\pi r^2$     (b)  $4\pi r^2$     (c)  $3\pi r^3$     (d)  $4\pi r^3$ .

**Problem 16.** Consider the polar curves  $r = 1$  and  $r = 1 - \sin \theta$ , for  $0 \leq \theta \leq 2\pi$ . The Cartesian coordinates of the intersections of these two curves are:

- (1)  $(x, y) = (-1, 0), (1, 0)$
- (2)  $(x, y) = (0, -1), (0, 1)$
- (3)  $(x, y) = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$
- (4)  $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ .

**Problem 17.** The area enclosed by one loop of the four-leaved rose  $r = \cos(2\theta)$  is

- (a)  $\pi$     (b)  $\frac{\pi}{2}$     (c)  $\frac{\pi}{4}$     (d)  $\frac{\pi}{8}$ .

**Problem 18.** The sequence  $\left\{ a_n = \frac{\sqrt{n}}{1 + \sqrt{n}} \right\}_{n=1}^{\infty}$  is

- (1) divergent
- (2) convergent to 0
- (3) convergent to  $-1$
- (4) convergent to 1.

**Problem 19.** The series  $\sum_{n=1}^{\infty} \frac{3n^2 + 2n + 1}{\sqrt{n^7 + 6n^5 + 3}}$  is

- (1) divergent
- (2) conditionally convergent
- (3) absolutely convergent
- (4) none of the above.

**Problem 20.** The series  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3 + 1}$

- (1) divergent
- (2) conditionally convergent
- (3) absolutely convergent
- (4) none of the above.