

NAME: .....TA (LAB SESSION): .....

**MATH 122 - CALCULUS II**  
**PRACTICE TEST I**

**Problem 1.** Find the area enclosed by the line  $y = x + 1$  and the parabola  $y^2 = 3x + 7$ .

**Problem 2.** Let  $S$  be the solid obtained by rotating about the  $y$ -axis the region bounded by  $y = 3 + 2x - x^2$  and  $x + y = 3$ .

- (1) Find the volume of  $S$  by evaluating an integral with respect to variable  $y$ .
- (2) Use the method of cylindrical shells to compute the volume of  $S$ .

**Problem 3.**

- (1) A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?
- (2) A spring has natural length of 20 cm. Compare the work  $W_1$  done in stretching the spring from 20 cm to 30 cm with the work  $W_2$  done in stretching it from 30 cm to 40 cm. How are  $W_1$  and  $W_2$  related?

**Problem 4.** Evaluate the following integrals:

(a)  $\int x^2 \ln x dx$     (b)  $\int e^{-x} \cos(2x) dx$     (c)  $\int \tan^3 x dx$     (d)  $\int \frac{x^3}{\sqrt{x^2 + 9}} dx$ .

**Problem 5.** Given the integral

$$\int_0^1 e^{x^2} dx.$$

- (1) Use Midpoint, Trapezoidal and Simpson rules with  $n = 5$  to approximate the given integral.
- (2) Estimate the errors of these approximations.
- (3) How large should  $n$  be taken to guarantee that the error is less than 0.00001?

**Problem 6.** Determine which of the following are improper integrals, and whether they are convergent.

(a)  $\int_3^7 \frac{1}{\sqrt{x^2 - 2x - 3}} dx$     (b)  $\int_0^{\pi/2} \sec x dx$     (c)  $\int_1^{\infty} \frac{1 + e^{-x}}{x} dx$     (d)  $\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6} dx$ .