

NAME: ..... TA (SECTION): .....

**MATH 122 - CALCULUS II**

**PRACTICE TEST II**

**Problem 1.** Let  $C$  be the arc of the parabola  $y = x^2$  from  $(1,1)$  to  $(2,4)$ .

- (1) Find the length of  $C$ .
- (2) Find the area of the surface of revolution obtained by rotating  $C$  about the  $y$ -axis.

**Problem 2.** Consider the initial-value problem

$$\begin{cases} y' = \frac{x^2}{y^2} \\ y(0) = 2. \end{cases}$$

- (1) Use Euler's method with step size  $h = 1$  to approximate  $y(3)$ .
- (2) Solve the given initial-value problem.

**Problem 3.** Solve the following initial-value problem:

$$\begin{cases} y' - 2xy = 3x^2e^{x^2} \\ y(0) = 5. \end{cases}$$

**Problem 4.** Let  $C$  be the curve with parametric equations:

$$\begin{cases} x = r(\theta - \sin \theta) \\ y = r(1 - \cos \theta), \end{cases} \quad \theta \in \mathbb{R}.$$

- (1) Find the tangent to  $C$  at the point where  $\theta = \pi/3$ .
- (2) At which points on  $C$  is the tangent horizontal? When is it vertical?
- (3) Find the arclength along  $C$  from  $\theta = \pi/3$  to  $\theta = 2\pi$ .

**Problem 5.** Let  $C$  and  $D$  be curves respectively given by polar equations

$$r = 1 + \cos \theta \text{ and } r = 1 - \cos \theta.$$

- (1) Find the Cartesian coordinates of the points on  $C$  and  $D$  where  $\theta = \pi/3$ .
- (2) Find the area of the region that lies inside both  $C$  and  $D$ .