LEWIS STRIKES AGAIN!

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The determinant evaluations:

Theorem 10 [K]:

$$\text{(1)} \qquad det\left[\frac{(x+y+i+j-1)!}{(x+2i-j)!(y+2j-i)!}\right]_{i,j}^{0,n-1}=\prod_{i=0}^{n-1}\frac{i!(x+y+i-1)!(2x+y+2i)_i(x+2y+2i)_i}{(x+2i)!(y+2i)!}$$

and

Theorem 8 [K]:

$$\text{(2)} \quad det\left[\frac{(x+y+i+j-1)!(y-x+3j-3i)}{(x+2i-j+1)!(y+2j-i+1)!}\right]_{i,j}^{0,n-1}=\prod_{i=0}^{n-1}\frac{i!(x+y+i-1)!(2x+y+2i+1)_i(x+2y+2i+1)_i}{(x+2i+1)!(y+2i+1)!}\times$$

$$\times\sum_{k=0}^{n}(-1)^k\binom{n}{k}(x)_k(y)_{n-k}$$

can be rewritten respectively as:

Theorem 10':

$$\text{(1')} \qquad det\left[\frac{(a+b+i+j-1)!}{(2a-b+2i-j)!(2b-a+2j-i)!}\right]_{i,j}^{0,m}=\prod_{i=0}^{m}\frac{i!(a+b+i-1)!(3a+3i-1)!(3b+3i-1)!}{(2a-b+2i)!(2b-a+2i)!(3a+2i-1)!(3b+2i-1)!}$$

and

Theorem 8':

(2')

$$det\left[\frac{(a+b+i+j-1)!(3b-3a+3j-3i)}{(2a-b+2i-j+1)!(2b-a+2j-i+1)!}\right]_{i,j}^{0,m}=\prod_{i=0}^{m}\frac{i!(a+b+i-1)!(3a+3i)!(3b+3i)!}{(2a-b+2i+1)!(2b-a+2i+1)!(3a+2i)!(3b+2i)!}\times$$

$$\times\sum_{k=0}^{m+1}(-1)^k\binom{m+1}{k}\frac{(2a-b+k-1)!(2b-a+m-k)!}{(2a-b-1)!(2b-a-1)!}$$

using the transformations: $x=2a-b, y=2b-a$ and $m=n-1$.

Now both (1') and (2') can be proved via *Lewis* [AE], [Z] :

$$X_m(a,b)=\frac{X_{m-1}(a,b)X_{m-1}(a+1,b+1)-X_{m-1}(a+1,b)X_{m-1}(a,b+1)}{X_{m-2}(a+1,b+1)}.$$

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

References

[AE] T. Amdeberhan, S.B. Ekhad, *A condensed condensation proof of a determinant evaluation conjectured by Greg Kuperberg and Jim Propp,* to appear in J. of Comb. Theory (A).

[K] C. Krattenthaler, *Determinant identities and a generalization of the number of totally symmetric self-complementary plane partitions,* preprint.

[Z] D. Zeilberger, *Reverend Charles to the aid of Major Percy and Fields Medalist Enrico*, Amer. Math. Monthly **103** (1996), 501-502.