LEWIS STRIKES AGAIN AND AGAIN!!

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Christian Krattenthaler asked if we can provide an easier proof of the determinant evaluation:

Theorem 1 [K]:

$$\begin{aligned} & det\left[\frac{(q;q)_{x+y+i+j-1}q^{-2ij}}{(q;q)_{x+2i-j}(q;q)_{y+2j-i}(-q^{x+y+1};q)_{i+j}}\right]_{i,j}^{0,n-1} \\ & = \prod_{i=0}^{n-1} q^{-2i^2}\frac{(q^2;q^2)_i(q;q)_{x+y+i-1}(q^{2x+y+2i};q)_i(q^{x+2y+2i};q)_i}{(q;q)_{x+2i}(q;q)_{y+2i}(-q^{x+y+1};q)_{n-1+i}}. \end{aligned} \tag{1}$$

Indeed, (1) can be rewritten as:

Theorem 1':

$$\begin{aligned} & det\left[\frac{(q;q)_{a+b+i+j-1}q^{-2(a+i)(b+j)}}{(q;q)_{2a-b+2i-j}(q;q)_{2b-a+2j-i}(-q;q)_{a+b+i+j}}\right]_{i,j}^{0,m} \\ & = \prod_{i=0}^{m} q^{-2(a+i)(b+i)}\frac{(q^2;q^2)_i(q;q)_{a+b+i-1}(q;q)_{3a+3i-1}(q;q)_{3b+3i-1}}{(q;q)_{2a-b+2i}(q;q)_{2b-a+2i}(q;q)_{3a+2i-1}(q;q)_{3b+2i-1}(-q;q)_{a+b+i+m}} \end{aligned} \tag{1'}$$

using the transformations: $x = 2a - b, y = 2b - a$ and $m = n - 1$.

Now (1') can be proved via *Lewis* [AE], [Z] :

$$X_m(a,b) = \frac{X_{m-1}(a,b)X_{m-1}(a+1,b+1) - X_{m-1}(a+1,b)X_{m-1}(a,b+1)}{X_{m-2}(a+1,b+1)}.$$

References

[AE] T. Amdeberhan, S.B. Ekhad, *A condensed condensation proof of a determinant evaluation conjectured by Greg Kuperberg and Jim Propp,* to appear in J. of Comb. Theory (A).

[K] C. Krattenthaler, *Some q-analogues of determinant identities which arose in plane partition enumeration,* preprint.

[Z] D. Zeilberger, *Reverend Charles to the aid of Major Percy and Fields Medalist Enrico*, Amer. Math. Monthly **103** (1996), 501-502.