

PROOF OF FORMULA 3.352.1

$$\int_0^a \frac{e^{-\mu x}}{x+b} dx = e^{\mu b} [\text{Ei}(-(a+b)\mu) - \text{Ei}(-b\mu)]$$

The exponential integral is defined by

$$\text{Ei}(x) = - \int_{-x}^{\infty} \frac{e^{-t}}{t} dt.$$

The change of variable $t = x + b$ gives

$$\int_0^a \frac{e^{-\mu x}}{x+b} dx = e^{\mu b} \int_b^{a+b} \frac{e^{-\mu t}}{t} dt.$$

Now write

$$\int_b^{a+b} \frac{e^{-\mu t}}{t} dt = \int_0^{a+b} \frac{e^{-\mu t}}{t} dt - \int_0^b \frac{e^{-\mu t}}{t} dt,$$

and let $s = \mu t$ to produce the result.