

PROOF OF FORMULA 3.361.2

$$\int_0^{\infty} \frac{e^{-qx}}{\sqrt{x}} dx = \sqrt{\frac{\pi}{q}}$$

Let $x = t^2$ to obtain

$$\int_0^{\infty} \frac{e^{-qx}}{\sqrt{x}} dx = 2 \int_0^{\infty} e^{-qt^2} dt.$$

The change of variables $t = v/\sqrt{q}$ gives

$$2 \int_0^{\infty} e^{-qt^2} dt = \frac{2}{\sqrt{q}} \int_0^{\infty} e^{-v^2} dv.$$

The result follows now from the value

$$\int_0^{\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}.$$