

PROOF OF FORMULA 3.362.2

$$\int_0^\infty \frac{e^{-\mu x} dx}{\sqrt{x+b}} = \sqrt{\frac{\pi}{\mu}} e^{\mu b} (1 - \operatorname{erf}(\sqrt{\mu b}))$$

Let $t = \sqrt{x+b}$ to obtain

$$\int_0^\infty \frac{e^{-\mu x} dx}{\sqrt{x+b}} = 2e^{\mu b} \int_{\sqrt{b}}^\infty e^{-\mu t^2} dt.$$

The scaling $s = \sqrt{\mu}t$ produces

$$\int_0^\infty \frac{e^{-\mu x} dx}{\sqrt{x+b}} = \frac{2e^{\mu b}}{\sqrt{\mu}} \int_{\sqrt{\mu b}}^\infty e^{-s^2} ds.$$

The final answer comes from the relation

$$\int_a^\infty e^{-s^2} ds = \frac{\sqrt{\pi}}{2} (1 - \operatorname{erf}(a)).$$