PROOF OF FORMULA 3.363.1

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x}\, dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - \pi\sqrt{a}\,(1 - \text{erf}(\sqrt{a\mu}\,))$$

Let $x = a + t^2$ to obtain

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x}\, dx = 2e^{-a\mu} J,$$

with

$$J = \int_0^\infty \frac{t^2 e^{-\mu t^2}\, dt}{a+t^2} = \int_0^\infty e^{-\mu t^2}\, dt - a \int_0^\infty \frac{e^{-\mu t^2}\, dt}{a+t^2}.$$

Therefore $J = \sqrt{\pi}/2\sqrt{\mu} - aK$, where

$$K = \int_0^\infty \frac{e^{-\mu t^2}\, dt}{a+t^2}.$$

Let $t = r/\sqrt{\mu}$ to obtain

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x}\, dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - 2a\sqrt{a} e^{-a\mu} \int_0^\infty \frac{e^{-r^2}\, dr}{r^2 + a\mu}.$$

The change of variables $s = r^2 + a\mu$ gives

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x}\, dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - a\sqrt{\mu} \int_b^\infty \frac{e^{-s}\, ds}{s\sqrt{s-b}},$$

with $b = a\mu$. Let $s = b\tau$ to obtain

$$\int_b^\infty \frac{e^{-s}\, ds}{s\sqrt{s-b}} = \frac{1}{\sqrt{b}} \int_1^\infty \frac{e^{-b\tau}\, d\tau}{\tau\sqrt{\tau - 1}}.$$

The integral obtained by differentiating with respect to b appears as entry **3.362.1**. Integrating back yields the result.