PROOF OF FORMULA 3.363.2

$$\int_a^\infty \frac{e^{-\mu x}\, dx}{x\sqrt{x-a}} = \frac{\pi}{\sqrt{a}} \left(1 - \mathrm{erf}(\sqrt{a\mu}\,)\right)$$

Let $x = at$ to obtain

$$\int_a^\infty \frac{e^{-\mu x}\, dx}{x\sqrt{x-a}} = \frac{1}{\sqrt{a}} \int_1^\infty \frac{e^{-bt}\, dt}{t\sqrt{t-1}},$$

with $b = \mu a$. Let $J(b)$ be the last integral, then

$$\frac{\partial J}{\partial b} = -\int_1^\infty \frac{e^{-bt}\, dt}{\sqrt{t-1}} = -\frac{\sqrt{\pi}}{\sqrt{b}} e^{-b},$$

according to **3.362.1**. Therefore

$$J(b) = J(0) - \sqrt{\pi} \int_0^b \frac{e^{-c}\, dc}{\sqrt{c}}.$$

The value $J(0) = \pi$ can be obtained by the change of variables $t = 1+1/v^2$. Finally, $c = u^2$ provides the value

$$J(b) = \pi(1 - \mathrm{erf}(\sqrt{b})).$$