PROOF OF FORMULA 3.369

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{(x+a)^{3/2}} = \frac{2}{\sqrt{a}} - 2\sqrt{\pi\mu}e^{a\mu}\left(1-\text{erf}(\sqrt{a\mu})\right)$$

The change of variables $t = (x+a)/\mu$ yields

$$\int_0^\infty \frac{e^{-\mu x}\,dx}{(x+a)^{3/2}} = \sqrt{\mu}e^{\mu a}\int_{\mu a}^\infty t^{-3/2}e^{-t}\,dt.$$

Integration by parts, writing $b = \mu a$ and making the change of variables $t = y^2$ produces

$$\int_{\mu a}^\infty t^{-3/2}e^{-t}\,dt = \frac{2e^{-b}}{\sqrt{b}} - 4\int_{\sqrt{b}}^\infty e^{-y^2}\,dy.$$

Replace and use

$$\text{erf}(a) = \sqrt{\frac{\pi}{2}}\int_0^a e^{-y^2}\,dy$$

to obtain the result. The identity

$$\int_0^a e^{-y^2}\,dy + \int_a^\infty e^{-y^2}\,dy = \frac{\sqrt{\pi}}{2},$$

is useful in the simplification of the answer.