PROOF OF FORMULA 3.382.1

$$\int_0^a (a-x)^{\nu} e^{\mu x} \, dx = \mu^{-\nu-1} e^{a\mu} \gamma(\nu+1, a\mu)$$

The function appearing in the answer is the *incomplete gamma function*

$$\gamma(\alpha, x) := \int_0^x e^{-t} t^{\alpha - 1} \, dt,$$

defined in 8.350.1.

Let s = a - x to obtain

$$\int_0^a (a-x)^{\nu} e^{\mu x} \, dx = e^{a\mu} \int_0^a s^{\nu} e^{-s\mu} \, ds.$$

The change of variables $t = s\mu$ to obtain

$$\int_0^a (a-x)^{\nu} e^{\mu x} \, dx = e^{a\mu} \mu^{-\nu-1} \int_0^{a\mu} e^{-t} t^{\nu} \, dt.$$

The formula is established.