PROOF OF FORMULA 3.411.14

$$\int_0^\infty \frac{x^2 e^{-nx}}{1-e^{-x}}\, dx = 2\sum_{k=n}^\infty \frac{1}{k^3} = 2\left(\zeta(3) - \sum_{k=1}^{n-1}\frac{1}{k^3}\right)$$

Entry **3.411.7** states that

$$\int_0^\infty \frac{x^{\nu-1}\, e^{-\mu x}\, dx}{1-e^{-bx}} = \frac{\Gamma(\nu)}{b^\nu}\zeta\left(\nu, \frac{\mu}{b}\right).$$

The special case $\nu = 3$, $\mu = n$ and $b = 1$ give

$$\int_0^\infty \frac{x^2\, e^{-nx}\, dx}{1-e^{-x}} = \Gamma(3)\zeta(3,n).$$

The answer comes from $\Gamma(3) = 2$ and

$$\zeta(3,n) = \sum_{k=0}^\infty \frac{1}{(k+n)^3} = \sum_{k=n}^\infty \frac{1}{k^3}.$$