

PROOF OF FORMULA 3.411.24

$$\int_{-\infty}^{\infty} \frac{x e^{\mu x} dx}{e^{\nu x} - 1} = \left(\frac{\pi}{\nu \sin \frac{\pi \mu}{\nu}} \right)^2$$

Entry **3.313.1** states that

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x} dx}{1 - e^{-x}} = \pi \cot \pi \mu.$$

The change of variables $x = \nu t$ gives

$$\int_{-\infty}^{\infty} \frac{e^{(1-\mu)\nu t} dt}{e^{\nu t} - 1} = \frac{\pi}{\nu} \cot \pi \mu.$$

This can be written as

$$\int_{-\infty}^{\infty} \frac{e^{\mu t} dt}{e^{\nu t} - 1} = -\frac{\pi}{\nu} \cot \frac{\pi \mu}{\nu}.$$

The result follows by differentiating with respect to the parameter μ .