

PROOF OF FORMULA 3.452.1

$$\int_0^{\infty} \frac{x dx}{\sqrt{e^x - 1}} = 2\pi \ln 2$$

The change of variables $t = e^{-x}$ gives

$$\int_0^{\infty} \frac{x dx}{\sqrt{e^x - 1}} = - \int_0^1 \frac{\ln t dt}{\sqrt{t} \sqrt{1-t}}.$$

Then $t = u^2$ produces

$$\int_0^1 \frac{\ln t dt}{\sqrt{t} \sqrt{1-t}} = 4 \int_0^1 \frac{\ln u du}{\sqrt{1-u^2}}.$$

This is evaluated in entry **4.241.7** to produce the result.