PROOF OF FORMULA 3.457.1

$$\int_0^\infty xe^{-x}(1-e^{-2x})^{n-1/2}\,dx = \frac{(2n-1)!!\,\pi}{4(2n)!!}\,(\gamma+\psi(n+1)+2\ln 2)$$

The change of variables $t = e^{-x}$ gives

$$\int_0^\infty xe^{-x}(1-e^{-2x})^{n-1/2}\,dx = -\int_0^1 (1-t^2)^{n-1/2}\,\ln t\,dt.$$

The result now follows from entry **4.241.5** which states that

$$\int_0^1 \sqrt{(1-x^2)^{2n-1}}\,\ln x\,dx = -\frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{4}\left(2\ln 2 + \sum_{k=1}^{n}\frac{1}{k}\right).$$

This form is equivalent to the one stated here.