

PROOF OF FORMULA 3.464

$$\int_0^\infty (e^{-\mu x^2} - e^{-\nu x^2}) \frac{dx}{x^2} = \sqrt{\pi}(\sqrt{\nu} - \sqrt{\mu})$$

Fix ν and define

$$f(\mu) = \int_0^\infty (e^{-\mu x^2} - e^{-\nu x^2}) \frac{dx}{x^2}.$$

Then

$$f'(\mu) = - \int_0^\infty e^{-\mu x^2} dx = -\frac{\sqrt{\pi}}{2} \mu^{-1/2},$$

and integrating back gives $f(\mu) = -\sqrt{\pi\mu} + C$. The constant C is determined from the boundary condition $f(\nu) = 0$.