PROOF OF FORMULA 3.468.1

$$\int_{u\sqrt{2}}^{\infty} \frac{e^{-x^2}\,dx}{x\,\sqrt{x^2-u^2}} = \frac{\pi}{4u}(1-\operatorname{erf}(u))^2$$

Let $x = ut$ and then $t = \sec\varphi$ to obtain

$$\int_{u\sqrt{2}}^{\infty} \frac{e^{-x^2}\,dx}{x\,\sqrt{x^2-u^2}} = \frac{e^{-u^2}}{u}\int_{\pi/4}^{\pi/2} e^{-u^2\tan^2\varphi}\,d\varphi.$$

Denote the integral by $f(u)$ and differentiate to get

$$f'(u) = -2u\int_{\pi/4}^{\pi/2} e^{-u^2\tan^2\varphi}\sec^2\varphi\,d\varphi + 2uf(u).$$

Denote this last integral by $g(u)$ and let $s = \tan\varphi$ to obtain

$$g(u) = \frac{\sqrt{\pi}}{2u}\left[1-\operatorname{erf}(u)\right],$$

so that

$$f'(u) = -\sqrt{\pi}(1-\operatorname{erf}(u)) + 2uf(u).$$

This can be written as

$$\frac{d}{du}\left[f(u)e^{-u^2}\right] = -\sqrt{\pi}e^{-u^2}(1-\operatorname{erf}(u)) = \frac{\pi}{4}\frac{d}{du}(1-\operatorname{erf}(u))^2.$$

Integrate and use the initial condition $f(0) = \pi/4$ to obtain the result.