# PROOF OF FORMULA 3.527.6

$$\int_0^\infty \frac{x^{\mu-1}\sinh ax}{\cosh^2 ax}\,dx = \frac{2\Gamma(\mu)}{a^\mu}\sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{\mu-1}}$$

Let $t = ax$ and write the integral as

$$\int_0^\infty \frac{x^{\mu-1}\sinh ax}{\cosh^2 ax}\,dx = \frac{2}{a^\mu}\int_0^\infty t^{\mu-1}\left(e^t - e^{-t}\right)e^{-2t}\frac{dt}{(1+e^{-2t})^2}.$$

Expand the integrand in a power series and separate the terms $e^t$ and $e^{-t}$ to obtain

$$\begin{aligned}\int_0^\infty \frac{x^{\mu-1}\sinh ax}{\cosh^2 ax}\,dx &= \frac{2}{a^\mu}\sum_{k=0}^\infty(-1)^k(k+1)\int_0^\infty t^{\mu-1}e^{-(2k+1)t}\,dt \\ &- \frac{2}{a^\mu}\sum_{k=0}^\infty(-1)^k(k+1)\int_0^\infty t^{\mu-1}e^{-(2k+3)t}\,dt.\end{aligned}$$

Scale the exponents $(2k+1)t$ and $(2k+3)t$ and shift the second sum to produce

$$\int_0^\infty \frac{x^{\mu-1}\sinh ax}{\cosh^2 ax}\,dx = \frac{2\Gamma(\mu)}{a^\mu}\left[1 + \sum_{k=1}^\infty \frac{(-1)^k(k+1)}{(2k+1)^\mu} - \sum_{k=1}^\infty \frac{(-1)^{k+1}k}{(2k+1)^\mu}\right]$$

This simplifies to give the result.