PROOF OF FORMULA 3.541.1

$$\int_0^\infty e^{-\mu x} \sinh^\nu bx \, dx = \frac{1}{2^{\nu+1} b} B\left(\frac{\mu}{2b} - \frac{\nu}{2}, \nu+1\right)$$

Start with

$$\sinh bx = \frac{e^{bx} - e^{-bx}}{2} = \frac{1 - e^{-2bx}}{2e^{-bx}}$$

and use the change of variables $t = e^{-2bx}$ to produce

$$\int_0^\infty e^{-\mu x} \sinh^\nu bx \, dx = \frac{1}{2^{\nu+1} \, b} \int_0^1 t^{\frac{\mu}{2b} - \frac{\nu}{2} - 1} (1-t)^\nu \, dt.$$

The result follows from the integral representation of the beta function

$$B(u,v) = \int_0^1 t^{u-1} (1-t)^{v-1} \, dt.$$