PROOF OF FORMULA 3.625.3

$$\int_0^{\pi/4} \frac{\sin^{2n-1} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m} x} \, dx = \frac{(2n-2)!! \, (2n-1)!!}{(2n+2m-1)!!}$$

The change of variables $t = \tan x$ gives

$$\int_0^{\pi/4} \frac{\sin^{2n-1} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m} x} \, dx = \int_0^1 t^{2n-1}(1-t^2)^{m-1/2} \, dt.$$

The change of variables $v = t^2$ gives

$$\int_0^{\pi/4} \frac{\sin^{2n-1} x \, \cos^{m-1/2} 2x}{\cos^{2n+2m} x} \, dx = \frac{1}{2} \int_0^1 v^{n-1}(1-v)^{m-1/2} \, dv = \frac{1}{2} B(n, m+1/2).$$

Now use

$$B(n, m+\tfrac{1}{2}) = \frac{\Gamma(n)\Gamma(m+1/2)}{\Gamma(n+m+1/2)}$$

and

$$\Gamma(m+1/2) = \frac{\sqrt{\pi}(2m-1)!!}{2^m}$$

to obtain the result.