PROOF OF FORMULA 3.662.1

$$\int_0^{\pi/2} (\sec x - 1)^\mu \, \sin x \, dx = \int_0^{\pi/2} (\, \operatorname{cosec} \, x - 1)^\mu \, \cos x \, dx = \frac{\pi \mu}{\sin \pi \mu}$$

The proof is given for the first integral. The second one is reduced to the first one by $x \mapsto \pi/2 - x$.

Let $t = \cos x$ to obtain

$$\int_0^{\pi/2} (\sec x - 1)^\mu \, \sin x \, dx = \int_0^1 t^{-\mu} (1 - t)^\mu \, dt.$$

This last integral is

$$B(1 - \mu, 1 + \mu) = \Gamma(1 - \mu, 1 + \mu) = \mu \Gamma(\mu) \Gamma(1 - \mu),$$

and this reduces to the stated answer.