PROOF OF FORMULA 3.757.1

$$\int_0^\infty \frac{\sin(ax)}{\sqrt{x}}\,dx = \sqrt{\frac{\pi}{2a}}$$

Let $t = ax$ to obtain

$$\int_0^\infty \frac{\sin(ax)}{\sqrt{x}}\,dx = \frac{1}{\sqrt{a}}\int_0^\infty \frac{\sin t}{\sqrt{t}}\,dt.$$

The change of variables $t = u^2$ yields

$$\int_0^\infty \frac{\sin t}{\sqrt{t}}\,dt = 2\int_0^\infty \sin(u^2)\,du.$$

Entry **3.691.1** states that this integral is $\sqrt{\pi}/2\sqrt{2}$. This establishes the formula.