PROOF OF FORMULA 4.212.4

$$\int_0^1 \frac{dx}{(a - \ln x)^2} = \frac{1}{a} + e^a \text{Ei}(-a)$$

Let $t = a - \ln x$ to obtain

$$\int_0^1 \frac{dx}{(a - \ln x)^2} = e^a \int_{-\infty}^{-a} \frac{e^t \, dt}{t^2}.$$

Now observe that

$$\frac{d}{dt}\left(\frac{e^t}{t}\right) = -\frac{e^t}{t^2} + \frac{e^t}{t},$$

and integrating from $-\infty$ to $-a$ yields

$$\frac{e^{-a}}{a} = \int_{-\infty}^{-a} \frac{e^t \, dt}{t^2} + \int_{-\infty}^{-a} \frac{e^t \, dt}{t}.$$

This gives

$$e^a \int_{-\infty}^{-a} \frac{e^t \, dt}{t^2} = \frac{1}{a} + e^a \int_{-\infty}^{-a} \frac{e^t \, dt}{t},$$

and the result follows.