PROOF OF FORMULA 4.224.1

$$\int_0^u \ln\sin x\,dx = L\left(\frac{\pi}{2}-u\right) - L\left(\frac{\pi}{2}\right)$$

The function L is defined by

$$L(u) = -\int_0^u \ln\cos t\,dt.$$

The change of variables $t = \pi/2 - x$ gives

$$\begin{aligned}\int_0^u \ln\sin x\,dx &= \int_{\pi/2-u}^{\pi/2} \ln\cos t\,dt \\ &= \int_0^{\pi/2} \ln\cos t\,dt - \int_0^{\pi/2-u} \ln\cos t\,dt.\end{aligned}$$

This gives the result.