PROOF OF FORMULA 4.227.12

$$\int_0^{\pi/2} \ln\left[(1-\tan x)^2\right]\,dx = \frac{\pi}{2}\ln 2 - 2G$$

The integral is

$$\int_0^{\pi/2} \ln\left[(1-\tan x)^2\right]\,dx = 2\int_0^{\pi/4} \ln(1-\tan x)\,dx + 2\int_{\pi/4}^{\pi/2} \ln(\tan x - 1)\,dx.$$

The change of variables $t = \pi/2 - x$ in the second integral yields

$$\int_0^{\pi/2} \ln\left[(1-\tan x)^2\right]\,dx = 4\int_0^{\pi/4} \ln(\cos x - \sin x)\,dx - 2\int_0^{\pi/4} \ln\cos x\,dx - 2\int_0^{\pi/4} \ln\sin x\,dx.$$

The result now follows from entries **4.225.1**, **4.224.5** and **4.224.2** that give

$$\int_0^{\pi/4} \ln(\cos x - \sin x)\,dx = -\frac{\pi}{8}\ln 2 - \frac{G}{2},$$

$$\int_0^{\pi/4} \ln\cos x\,dx = -\frac{\pi}{4}\ln 2 + \frac{G}{2},$$

$$\int_0^{\pi/4} \ln\sin x\,dx = -\frac{\pi}{4}\ln 2 - \frac{G}{2},$$

respectively.