PROOF OF FORMULA 4.234.6

$$\int_0^\infty \frac{\ln x \, dx}{(a^2+b^2x^2)(1+x^2)} = \frac{\pi b}{2a(b^2-a^2)} \ln \frac{a}{b}$$

The partial fraction decompositon

$$\frac{1}{(a^2+b^2x^2)(1+x^2)} = \frac{1}{(a^2-b^2)(1+x^2)} - \frac{b^2}{a^2-b^2} \frac{1}{a^2+b^2x^2}.$$

Therefore

$$\int_0^\infty \frac{\ln x \, dx}{(a^2+b^2x^2)(1+x^2)} = \frac{1}{a^2-b^2} \int_0^\infty \frac{\ln x \, dx}{1+x^2} - \frac{b^2}{a^2-b^2} \int_0^\infty \frac{\ln x \, dx}{a^2+b^2x^2}.$$

The second integral is given in entry **4.231.8** as

$$\int_0^\infty \frac{\ln x \, dx}{a^2+b^2x^2} = \frac{\pi}{2ab} \ln \frac{a}{b}.$$

The special case $a = b = 1$ shows that the first integral vanishes. This gives the result.